MAXWELL'S PARADOX:
THE METAPHYSICS OF
CLASSICAL ELECTRODYNAMICS AND
ITS TIME REVERSAL INVARIANCE

Valia Allori

Abstract

In this paper I argue that recent discussion on the time-reversal invariance of classical electrodynamics rests on the fact that the theories compared are different theories. If so, the controversy will not be resolved until we have established which alternative is the most natural. It turns out that we have a paradox, namely that the following three claims are incompatible: the electromagnetic fields are real, classical electrodynamics is time reversal invariant, and the content of the state of affairs of the world does not depend on whether it belongs to a forward or a backward sequence of states of the world.

Abstract word count: 101; Total word count: 4569

Keywords: Classical Electrodynamics; Time Reversal Invariance.

1. Introduction

A recent disagreement among philosophers of physics revolves around the question of whether classical electrodynamics (CED) is time reversal invariant. David Albert [Albert 2000] argues that CED violates time reversal invariance, while orthodoxy has been defended, among others, by Frank Arntzenius [Arntzenius 2004], John Earman [Earman 2002] and David Malament [Malament 2004]. Paul Horwich [Horwich 1987] instead has put forward an intermediate position, which has been taken to be incoherent [Arntzenius 2004]. I think that the analysis provided by the literature does not get us to the heart of the matter, and does not account for Horwich's position, while instead my account can. I believe that different judgments about time reversal invariance of CED rest on different judgments about what the ontology of CED is. If so, we cannot settle the dispute over the time reversal invariance of CED until we have established which alternative is the most natural way to interpret the formalism of CED. It turns out that each alternative CED is so costly that the situation could be described as a paradox, which I call "Maxwell's Paradox:" assuming that a possible history of the world is a sequence of instantaneous states, it is impossible to simultaneously hold that electromagnetic fields are just as real as particles, that the ontology of the theory does not depend on whether we are considering the forward or the backward history of the world, and that CED is time reversal invariant.

In Section 2 I present Albert's argument against the time reversal invariance of CED, while in Section 3 I discuss the defense of the traditional position. In Section 4 I argue that the different proposals are different takes on the ontology of CED. An analysis of the merits and the objections of each of these theories are provided in Section 5, and in Section 6 I formulate Maxwell's paradox.

2. The Argument against Invariance

According to Albert [Albert 2000] a complete description of the world, the instantaneous state of the world S, has to be genuinely instantaneous and complete. Typically, in classical mechanics (CM) the state is taken to be constituted by the couple of positions and velocities. Albert instead thinks that such couple should be called the dynamical condition D at an instant: it violates independence and provides all the information required "in order to bring the full predictive resources of the dynamical laws of physics to bear." Rephrasing, the instantaneous state S represents what exists in the world at one instant, and the dynamical condition D specifies what is needed at one time to determine the state of the system at another time. In CM, S is given by the particles' positions, while D by the positions and the velocities.

A time reversal transformation involves an operator T that transforms a possible temporal sequence of instantaneous states $S_1, S_2,\ldots,S_N$ (a possible history of the world), into the backward sequence $T(S_1, S_2,\ldots,S_N)= S_N, S_{N-1},\ldots,S_1$. According to the traditional notion of time reversal invariance, a theory is time reversal invariant if and only, for each possible history of the world $S_1, S_2,...,S_N$, also the backward sequence $S_N, S_{N-1},...,S_1$ is a possible history of the world. A history of the world can be thought as a movie, in which the sequence of the various instantaneous states is the sequence of the different frames. To say that a theory is time reversal invariant is then to say that the movie projected backward and the movie projected forward both represent possible state of affairs of the world. Albert claims that S should be invariant under T: S represents what there is in the world, and a sequence of S represents a possible history of the world, so remembering the movie analogy, T is acting on the order of the frames, not on their content, which remains the same.

In CM, S remains invariant under T: T(x)=x. The velocity instead flips sing, since it transform under T according to its definition as a function of x and t: T(v)=T(dx/dt)=dx/(-dt) =-dx/dt =-v. Therefore, while S is left unchanged, D transforms as T(x,v)=(x,-v). CM is time reversal invariant because both the forward history x(t) and the backward history x(-t) are possible histories of the world.

When considering CED, Albert believes we need to add to S also "the magnitudes and directions of the electric (E) and magnetic (B) fields at every point in space." That is, S is given by the triple (x,E,B). The fields are, unlike velocities, logically independent of the particles' positions and therefore they should be added to S in order to complete the picture of the world at one time. That is, electromagnetic fields are real just as much as the particles are. Albert claims that S should be invariant under T. After all, there is no reason why the fields should change under T, Albert says: the velocity is defined as the rate of change of position, so that it makes sense for it to flip sign under T, but the fields have no such definition. So, he argues, they should be represented simply by vector functions, which would not transform under T. As a matter of fact, though, in order for CED to be time reversal invariant we need T(E)=E and T(B)= - B: given a possible history of the world $S_1, S_2,...,S_N$ then $S'_N, S'_{N-1},...,S'_1$ (where S'=(x,E,-B) for any time) is also a possible history of the world, $S_N, S_{N-1},...,S_1$ is not. This is incompatible with the requirement above, so CED is not time reversal invariant.

It is useful to spell out Albert's argument schematically as follows:

1. [FIELDS]: in CED electromagnetic fields belong to the instantaneous state. That is S=(x,E,B);
2. [TIME REVERSAL]: a time reversal transformation is one that turns a sequence of states of the world $S_1, S_2,...,S_N$ into its reverse $S_N, S_{N-1},...,S_1$;
3. [T-R INVARIANCE]: a theory is time reversal invariant just in case the (theoretical) forward and backward sequences of states of the world both correctly describe possible states of affairs of the world;
4. In particular, for CED to be time reversal invariant we need B to flip sign under T. That is, T(B)= - B;

5. [STATE]: under a time reversal transformation the instantaneous state of the world at one time does not change. That is, T(S)=S;
6. In particular, T(B)=B;
7. 6 contradicts 4.
8. Therefore CED is not time reversal invariant.

Albert judges [FIELDS], [TIME REVERSAL], [T-R INVARIANCE], [STATE] and the fact that there is no reason for B to flip sign under T to be so obviously true that we have to reject that CED is time reversal invariant. Clearly, this is not the only possibility: one could reject one (or more) of the other premises. I argue that the disagreement about the time reversal invariance of CED can be accounted for focusing on [FIELDS] and [STATE], and this is fundamentally a disagreement about ontology.

3. The Defense of Invariance

Arntzenius [Arntzenius 2004], Earman [Earman 2002] and Malament [Malament 2004], among others, disagree with Albert's conclusion. I will only discuss the results of Malament, but since Earman and Arntzenius provide similar analyses, I will call their account 'AEM.' Malament's main goal is to show how one can naturally define the electromagnetic fields so that they transform under T as required in order to make CED time reversal invariant. We represent a particle's world-line as a smooth curve on space-time, and its 4-velocity by the tangent to such curve at any point. Imposing certain constraints, one obtains that the mathematical object that correctly represent the electromagnetic force is an antisymmetric tensor. One can similarly define the electromagnetic currents. From them one can recover all CED: the electromagnetic fields and Maxwell's equations. It turns out that the electric field E is mathematically represented by a polar vector and the magnetic field B by an axial vector. This explains why B flips sign under T and E does not: by definition a polar (true) vector will match its mirror image under reflection, while an axial (pseudo) vector under reflection will also match in magnitude its mirror image but it will be in the opposite direction.

To be able to properly compare Albert's approach to the one of AEM, we need to clarify what AEM take to be the components of S, and what their concept of time reversal transpiration is. The first point is easy: AEM seem to take the fields as part of S[1]. The first point is easy: AEM seem to take the fields as part of S. Regarding the notion of time reversal, the situation is a little more complex. Jill North [North 2008] has argued that AEM's conception of time reversal is different from the one of Albert. According to North, Albert's idea of time reversal transformation T_A "mirrors the material content of space--time across a time slice," while Malament's time reversal transformation T_M "inverts the temporal orientation." The difference between the two, she argues, is that in Malament's account the temporal orientation is considered alike to

[1] Earman and Arntzenius explicitly hold that, while Malament is not so explicit about it.

a physical field, so that some field τ should also be added to S. If North is correct, then AEM reject [TIME REVERSAL], and [STATE] fails by definition: τ is in S and T_M flips it. I think that, regardless of whether there is a difference in the time reversal transformation, the fundamental disagreement between Albert and Malament is fundamentally about [STATE]. In fact, if they both accept [TIME REVERSAL], Malament, who defines B as an axial vector, has to deny [STATE] to get time reversal invariance of CED. If instead Malament denies [TIME REVERSAL], then he would deny [STATE] as well, this time as a consequence of its choice of time reversal: τ is in S and τ flips sign under T (by definition of $T=T_M$). So, AEM conclude CED is time reversal invariant because they assume [FIELDS] but reject that there is no reason for B to flip sign under T, since B is an axial vector. Because of this, AEM reject [STATE]: B belongs to S, and S will not remain invariant under T.

In contrast to the analysis above, Stephen Leeds [Leds 2006] has argued that the disagreement is due to a mathematical mistake. Malament's analysis is in terms of passive transformations, while Albert's account is in terms of active ones. When we consider the active counterparts of the passive transformations used by Malament, we discover they do not provide the correct transformation for B. I think this conclusion is wrong: the problem is that in his derivation of the active counterparts of Malament's transformations, Leeds uses the constraint that S remains invariant under T, which, as we just saw, Malament denies.

Least but not last we have Horwich [Horwich 1987], who seems to accept both [STATE] and the time reversal invariance of CED. He develops this position in just few sentences, so how one can have an invariant CED with an invariant S is up for debate. While Arntzenius thinks that Horwich is simply incoherent [Arntzenius 2004], presumably because he assumes Horwich accepts [FILEDS] as well. Instead, I will argue that Horwich accepts [STATE] but he rejects [FIELDS], so that he can consistently claim CED is time reversal invariant.

4. The Possible Metaphysics of Classical Electrodynamics

I believe people disagree about time reversal of CED because they disagree about ontology. My main claim is that CED can be consistently interpreted as depicting different worlds by Albert and by AEM: the world is made of particles and fields in both cases, but such fields are mathematically represented by different objects, vector functions for Albert, axial vectors for the others. The symmetry properties of a theory depend on the ontology of the theory: different ontologies are captured by different mathematical objects, which have different properties. Since AEM and Albert disagree about what kinds of object fields are, they disagree about what symmetry properties the theory has. Contrary to Albert, AEM can accept the time reversal invariance of CED because they reject [STATE]: an axial vector will not stay invariant under T. Therefore, when they argue about the time reversibility of CED, they actually talk past each other:

they are talking about two different CEDs, and it is not unexpected that different theories have different symmetry properties.

It is not surprising one could disagree on how to interpret a theory: any physical theory is expressed in terms of mathematical relations among different variables. In order to interpret a theory realistically, one needs to take at least some of these variables as representing physical objects. However, the very same mathematical framework could be interpreted in different ways, and certain interpretations might be more natural than others: there is, so to speak, a sort of underdetermination of the ontology by the formalism. Given its definition, S captures the metaphysics of the theory, while D contains also the variables needed to implement the dynamics for the stuff in S. To make this point also graphically, we can use the symbol ";". It will separate in D the elements that belong to S, which we will put on the left of the semicolon, from the rest of the variables in the theory. Interpreting the mathematical object in S in the 'most natural way' will give us what there is physically in the world. For example, in CM D=(x;v), and S=(x), which naturally represents point-particles. In CED one could argue about which is the most appropriate way of mathematically describing E and B, and each choice has consequences: the symmetry properties of a theory are the symmetry properties of the objects in its ontology, and different mathematical objects will behave differently under a given transformation. As a result, depending on what mathematical object we take to represent B, CED will or will not be time reversal invariant.

So, here is how one can summarize Albert's position and the one of AEM respectively[2]:

- Albert-CED=CED_{xEB}=(x,E,B;) -- the world is constituted by particles and fields, the latter being represented by vector functions. So, [FIELDS] and [STATE] hold, but the theory is not time reversal invariant.
- AEM-CED=$CED_{xEB'}$ =(x,E,B';) -- the world is made of particles and fields, the latter being represented by polar and axial vectors respectively. So, [FIELDS] is true, but [STATE] is not, allowing for the theory to be time reversal invariant[3].

The disagreement here is therefore about which is the most suitable way of mathematically describing a piece of furniture of the world, namely the electromagnetic fields: are they true or axial vectors? Of course, we could also disagree about what belongs to S. By moving the semicolon, we can generate different theories assigning a different role to the mathematical object of the same mathematical formalism. For instance, we could have $CM_{x,v}$=(x,v;), or CM_v=(v;x) in addition to the more natural CM_x=(x;v): $CM_{x,v}$ describes a world of particles' positions and velocities, while CM_v a world of only velocities. Note that the fact that there are different ways of reading the

---

[2] For sake of simplicity, we will not mention velocities.

[3] Here B' indicates that the mathematical object representing a magnetic field in this theory is different from the mathematical object that represents it in the theory above.

formalism does not mean that they are all sensible. For instance, CM is usually taken to be CM_x. And the reasons for this are the ones presented by Albert: in $CM_{x,v}$ S is not really instantaneous, while CM_v is not complete[4]. Observe that if we start moving the semicolon we obtain, among others, the following position:

- Horwich-CED=CED_x=(x;E,B′) -- the world is made of particles, while the fields do not compose matter: they are mathematical fictions to correctly describe the behavior of the particles. That is, [FIELDS] fails, but [STATE] holds, since B does not belong to S and therefore what B does under T is irrelevant and CED is time reversal invariant.

Horwich has been accused by Arntzenius to be confused: "Horwich infers that in the reversed history the magnetic field points in the opposite direction. But this is inconsistent with his claim that 'basic' or 'fundamental' quantities should not undergo time reversal operations" [Arntzenius 2004]. While if he is taken to endorse the theory discussed last the confusion dissolves: the fields do not belong to S, and therefore are not part of the basic and fundamental quantities.

5. Which Metaphysics?

All the proposed theories seem to be possible ways of metaphysically interpret the formalism of CED. They provide different pictures of the world, and accordingly have different symmetry properties. Albert, considering CED to be $CED_{x,E,B}$, judges it to break time reversal invariance; AEM, considering CED to be $CED_{x,E,B'}$ conclude the contrary; Horwich, arguably considering CED to be CED_x, considers it to be time reversal invariant but for a different reason. But which of the above is the true CED?

The main argument to favor $CED_{x,E,B'}$ over $CED_{x,E,B}$ lies on the importance of symmetry properties: $CED_{x,E,B'}$ is better than $CED_{x,E,B}$ because the former has more symmetry properties, and physicists always put a lot of weight on symmetries, since they seem to construct theories around symmetry groups. To defend Albert's position, the only choice seems to be to question their importance of symmetry properties, but it seems a difficult position to defend.

Other reasons to favor $CED_{x,E,B'}$ are much weaker than the one we just saw. One could argue that $CED_{x,E,B'}$ is coherent with history: we started off describing matter as made of particles; we then discovered electromagnetic phenomena and added fields to S; only in the relativistic framework we realized that the magnetic fields are mathematically represented by axial vectors, that is we arrived at $CED_{x,E,B'}$. A problem with this argument could be that when we discover a theory should not matter: maybe there are reasons to take the older theory more seriously than the most recent one. It seems to me, though, that this reply misses the point: the suggestion is that the most

---

[4] Arguably, an Aristotelian about properties would like CM_{xv} very much, since it describes a world in which we have both the objects *and* the velocities. This seems to be discussed also in [Arntzenius 2000].

recent theory is best not in virtue of being recent, but because it represents our true understanding of nature. Another argument for $CED_{x,E,B'}$ over $CED_{x,E,B}$ appeals to Ockham's razor [Arntzenius and Greaves 2009]: $CED_{x,E,B}$ needs a standard absolute rest and an objective temporal orientation, while $CED_{x,E,B'}$ does not. The potential problem with this is that one could argue that simplicity is not really a virtue.

Be that as it may, $CED_{x,E,B'}$ faces a serious challenge: in this theory S changes under T. That amounts to say that the content of a state of the world could change depending on whether it comes from the forward or the backward movie of the world. This seems at best counterintuitive, , and does not happen in Albert-CED and Horwich-CED: how is it possible that in the S from the forward movie the magnetic fields point in one direction, but in backward one it points to the opposite direction? One could take this to be a reductio ad absurdum for the theory. Arntzenius [Arntzenius 2004] tries to justify the rejection of [STATE] in terms of what he calls a 'geometrical' transformation: given a particular mathematical object, there is a natural way for it to transform under a particular transformation which depends on its geometrical intrinsic definition. That is, the electromagnetic fields are intrinsically defined as to transform under T to make the theory invariant, which makes [STATE] false. Even if correct, though, this does not help at all, especially since these fields are real: it still does not seem sensible that the field would be pointing in two different directions in the backward and forward movie. Another possible reply to this challenge, which strikes for its weakness, is to argue that this is just counterintuitive, but less counterintuitive than what the alternative positions propose.

The main arguments to prefer CED_x to the alternatives are the following. First, the symmetry argument could be used to favor CED_x over $CED_{x,E,B}$. Also, a simplicity argument could be used to favor CED_x over $CED_{x,E,B'}$: they both have symmetries but the former is better because it has a simpler ontology. If we can account for everything just assuming there are particles, the argument says, why also assume fields? The key, the argument continues, is that we don't need them in S: they can simply be regarded as playing a role in determining the motion of the particles, the argument concludes. In addition to the problem of using simplicity as a virtue, the worry with this kind of argument is that it does not seem to be true that we just need particles to explain the phenomena. In fact, there seems to be energy associated to the fields, and how can CED_x explain such energy if there are no fields? Another argument for CED_x over $CED_{x,E,B'}$ is that in the former S is invariant under T while in the latter S is not. This is the same as what happens in $CED_{x,E,B}$, but here we also have more symmetry properties.

A first worry about CED_x would be that asserting that there are no fields is contrary to our intuitions and our ordinary beliefs. That is, CED_x makes many of our beliefs false, and theories like that, as skepticism for instance, are not to be favored. One could respond rejecting such criterion of theory choice: after all, many of the modern scientific theories ask us to revise many of our ordinary beliefs, but we do not reject them because of this. At the same time, though, to select a counterintuitive theory is always problematical. Be that as it may, In addition to this genuine worry, there are others that actually seem just misunderstandings. First, one could think that in this theory S is not a true instantaneous state of the world: there are no fields, so it is incomplete. This would be misunderstanding the position, though: S is complete, since the theory says there are really no fields in the world. Another complaint against CED_x is that there are no free fields, that all fields are `generated' by particles. But if the fields are not supposed to represent physical objects, then the solutions of Maxwell's equations, including free solutions, never have any physical meaning. Another alleged challenge to CED_x rests on the fact that the theory is completely indeterministic[5]: if in S we just have particles' positions, then in the instantaneous state we do not have all that we need to determine the instantaneous state at a later time. But when we talk about determinism what we really have in mind involves D instead of S. That is, determinism as we understand it is defined in terms of D: a theory is deterministic just in case, given D at one time, one can determine D at a different time. According to this definition, neither CM nor CED_x are indeterministic theories.

6. Maxwell's Paradox

I have argued so far that Albert, Horwich, and AEM entertain distinct and consistent theories of CED. Since symmetry properties are determined by the ontology of the theory, we need to determine which of the previous proposals provides the most natural ontology: until we have solved the controversy about which alternative is the most natural, we will not be able to solve the disagreement about the symmetry properties of CED.

Unfortunately, as this overview has just shown, the discussion is far from being settled: each proposal has costs so high that we might even claim that we are in a presence of a paradox. Let us come back to Albert's original argument. If the definitions of time reversal transformation and invariance have not been truly questioned, then it can be re-written as follows:

1. [FIELDS]: Electromagnetic fields in CED are just as real as particles;
2. [STATE]: The content of a state of the world does not depend on whether this state is taken from a forward or a backward sequence of states;
3. Therefore, ~{T R INVARIANCE of CED]: CED is not time reversal invariant.

If we go with AEM, what we get is:

1. [FIELDS]: Electromagnetic fields in CED are just as real as particles;
2. [T R INVARIANCE of CED]: CED is time reversal invariant;
3. Therefore ~[STATE]: the content of a state of the world depends on whether this state is taken from a forward or a backward sequence of states.

[5] See [Arntzenius 2000] for a similar worry in the framework of CM.

Finally, if Horwich is correct we have:

1. [STATE]: The content of a state of the world does not depend on whether this state is taken from a forward or a backward sequence of states;
2. [T R INVARIANCE of CED]: CED is time reversal invariant;
3. Therefore ~[FIELDS]: electromagnetic fields in CED are not just as real as particles.

That means that we cannot take [FIELDS], [STATE] and [T R INVARIANCE of CED] to be true for the same theory. So, however we turn it, we are in presence of a very counterintuitive conclusion derived from apparently acceptable reasoning from apparently acceptable premises. The debaters can be taken to be struggling with this paradox: we have only three choices, which correspond to the accounts of our three players. Albert solves the paradox rejecting the time reversal invariance of CED, motivated by the intuitions that electromagnetic fields are real, and that the ordering of a sequence of states should not change the content of such states. But this is not a light cost: symmetries are important! AEM instead reject [STATE]: they want electromagnetic fields to constitute matter like Albert does, but they also want symmetries. So they have to give up the idea that the content of an instantaneous state of the world does not change depending on whether this state is taken from the forward or the backward story of the world. This, again, strikes are counterintuitive: how can the content of the state change under T? Finally, Horwich wants to keep the time reversal symmetry and the invariance of the content of the states with respect to time reversal, so he denies the reality of fields. But is it really an acceptable choice to say that there are no fields?

The lesson to be drawn from this discussion therefore seems to be that CED involves a paradox, which I will call Maxwell's paradox. Assuming the given definitions of time reversal transformation and invariance, the following very plausible claims are incompatible:

- Electromagnetic fields in CED are just as real as particles;
- The content of a state of the world does not depend on whether this state is taken from a forward or a backward sequence of states;
- CED is time reversal invariant.

No matter how much we would like to keep all three of them, we simply cannot.

References:

- [Albert 2000] Albert, David Z.: Time and Chance, Harvard University Press, Cambridge, Massachusetts (2000).
- [Arntzenius 2004] Arntzenius, Frank: Time Reversal Operation, Representation of the Lorentz Group and the Direction of Time, Studies in Histories and Philosophy of Modern Physics, 35 (1) (2004).
- [Arntzenius 2000] Arntzenius, Frank: Are there Instantaneous Velocities?, The Monist 83: 187-208 (2000).
- [Arntzenius and Greaves 2009] Arntzenius, Frank, and Greaves, Hillary: Time Reversal in Classical Electrodynamics. British Journal for the Philosophy of Science 60: 557-584 (2009).
- [Earman 2002] Earman, John: What Time Reversal invariance is and why it Matters.
- International Studies in the Philosophy of Science 16: 245-264 (2002).
- [Horwich 1987] Horwich, Paul: Asymmetries in Time: Problems in the Philosophy of Science. Cambridge, Mass: MIT Press (1987).
- [Leeds 2006] Leeds, Stephen: Discussion: Malament on Time Reversal. Philosophy of Science 73: 448-458 (2006).
- [Malament 2004] Malament, David: On the Time Reversal Invariance of Classical Electromagnetic Theory, Studies in Histories and Philosophy of Modern Physics 35B: 295-315 (2004).
- [North 2008] North, Jill: Two Views of Time Reversal. Philosophy of Science 75 2: 201-223 (2008).