Comment on
"YABLO'S PARADOX AND RUSSELLIAN PROPOSITIONS"
by Gregory Landini

In his ingenious paper Gregory argues that Yablo fails in his attempt to produce a paradox without self reference. Indeed, he fails to produce a paradox at all. I will first briefly describe Yablo's paradox and explain why it is supposed to be paradoxical, then I will present Greg's argument that Yablo fails to produce a paradox, and finally I will comment on it.

Yablo claims that self-reference is not necessary to create liar-like paradoxes. He in fact provides an example of a list of non self-referential sentences that are paradoxical in the same sense as the liar sentence is paradoxical. He proposes an infinite list of sentences, each of which is of the form: Sn: {for all k greater than n, Sk is false}. This list of sentences produces a paradox, Yablo claims, in the sense that there is no way to consistently assign truth values to each individual sentence of the list. Here it is the reason. Suppose all the sentences of the list are false. As a consequence, what S1 asserts should be true: in fact S1 asserts that all the sentences after S1 are false. But then we obtain contradiction since S1 being true is contrary to the original assumption that all sentences are false. Suppose instead that there is at least some sentence that is true: say Sn is true. That means that it is true that for all k greater than n we have that Sk is false. In particular Sn+1 is false, and all the Sk with k greater than n+1 are false as well. But this is sufficient to make Sn+1 true, which again leads to contradiction.
(Notice that if the list is not infinite, then here is no paradox. In fact, suppose that there are just 2 sentences: S1: {for all k greater than 1 Sk are false}, S2: { for all k greater than 2 Sk are false}. Here is a possible assignment of truth values: S2 false, S1 true. Here is why it is possible: if S1 is false, then all S2, S3, S4, S5 are true; in particular S2 is true, but no contradiction arises because S2 refers to sentences outside the list. Here is another possible truth value assignment: S1 false and S2 is true. Here is why it is possible: If S1 is true, then S2, S3, S4, S5 .... are false; in particular S2 is false,but no contradiction arises because S2 refers to sentences outside the list. Since it is possible to coherently assign truth value to individual sentences, a finite Yablo list is not paradoxical.)
This paradox is supposed to be liar-like because also in the case of the liar paradox we have a sentence that leads to contradiction when we try to determine whether it is true or false. The liar paradox in fact is produced considering a sentence that says of itself that it is false. So that if it is false, it is true and *vice versa*.
But, differently from what happens in the case of the liar, it is claimed by Yablo, his formulation does not appeal to self-reference. A statement is self-referent is it refers to itself, and this is seems what is happening in the liar sentence. Instead the Yablo list is not self-referential, Yablo says, because every sentence does not refer to itself but to the sentence that is going to be uttered next (Sn refers to all other sentences Sk with k greater than n). So, Yablo concludes, the infinite list above provides a non self-referential version of the liar sentence.

Greg argues that Yablo is mistaken in his claim: in fact he fails to produce a self-referential paradox because he fails to produce a paradox at all.
Greg claims that in order to get the Yablo-list we need to have a function from natural numbers to sentences that generates such a list. If a generating function cannot be identified, or is ill-defined, or does not exist then the list cannot be generated and the paradox cannot arise.
Greg identifies such function as the # function (that he writes as #n: "(∀k) (k>n .→ .~True # k)").
There are two things that are crucial to notice about #. The first is that it contains in itself the # sign, so that the function is self-referential. Second, in order to specify a function you need to specify a range and a domain, and the range for # contains also the instruction to use # itself. That is, syntax alone is not enough to define #, we need also some semantic. But that means that # cannot be defined with the

means at our disposal. And that means in turn that the function, not being definable, cannot start generating the list. In order to write the list we need to understand what # says, but in order to understand what # says we need to understand what # says! So how can we generate the list?
Then, Greg attempts to formulate Yablo's paradox in the frame of Russellian theory of propositions, but there is a similar obstacle. The conclusion is then the same as above: the function that generates the Yablo list, does not exist.

To sum up so far, I think that what is claimed here is the following:
1-In order to have Yablo's paradox we need to have an infinite list of sentences.
2-In order to have this infinite list, we need to have a generating function
3-A generating function should be definable only in terms of the (symbolic) language and not in terms of other outside helps, otherwise it does not exist
(this is the recognition that in order to define a function, one needs to provide a range and a domain within the powers of the language you are using)
4- If it is not possible to define such function, the paradox does not arise
5- The function is not suitably definable/does not exist
6- Therefore the paradox does not arise.

Even if I am sympathetic with Greg's attitude, here are some comments about his argument.
My main problem regards the claim that if it is not possible to define a generating function then the paradox does not arise. In fact I am not so sure I am totally convinced by this: why do we need a generating function to produce a paradox?
Greg's idea is that we need a generating function in order to have an infinite list of statements: since the list is infinite, we cannot write them all out, and therefore we need this function. It is true that we cannot write down in a finite time an infinite list. Nonetheless, why it would be needed to have the entire list written down? After all, when I explain Yablo's paradox to someone, I do not write down whole the list at all.
In reply, one could claim that I can use only a finite list to explain the paradox because I understand how to go on writing all the other sentences in the list, and the reason for this is that I rely (more or less implicitly) on a generating function: I have the generating function in mind, that's why I can write down the sentences one after the other, and that is why I understand there is a paradox even if there is a finite list.
But this seems to be an argument that the generating function exists (given that I am using it to write the sentences and to understand what's going on).
Greg would say that I tend to make this inference about the existence of the function because I think there is a paradox to start with: the paradox is a consequence of the fact that there is an infinite list, and the infinite list exists only if there is a generating function; since there is a paradox, there has to be a function. While Greg would go the other way: since there is no generating function, there is no infinite list, there is no paradox, it just *seems* there is one.
Even if I understand the strategy, I have the intuition that that cannot be right: in the moment I write some sentence on the list there is some rule that I am following, right? Yes: otherwise, how could I write it? So maybe one could argue that one does not need a generating function at all to get an infinite list: while having a generating function allows us to generate an infinite list, it is not the only way.
Here is what I mean. Greg reads "Sk" in the definition of Sn:{for all k greater than n then Sk is false} as "the sentence whose number under the function generating the list is k". With this reading, then of course you have the need of a generating function. But why not simply stick to a natural reading of Sn plus a couple of other rules? A natural reading of Sn is the following: "all the sentences that I am going to write below this very sentence that I am writing now are false". Then add the following rules: rule 1: write the first sentence, rule 2: keep on writing.

In this way we do generate an infinite list, without the need of a function that needs to be formally defined in the language. So even if the generating function does not exist, we still have the infinite list, and with that we also have the paradox. Notice though that in this way we have produced a self-referential paradox: in our reading of Sn, Sn refers to Sn. (This reading does indeed make Yablo's paradox much closer to the usual liar paradox than the original formulation.) But one thing is to say that Yablo has failed in producing a paradox that is not self-referential because there is no paradox, and another is to say that he failed because the paradox is self-referential.
In my opinion, therefore, what should be concluded here is that even if there is no generating function for Yablo's list, that does not mean we are not in presence of a paradox: we could have generated the list with the reading and the rules above. They fail to be properly definable as a generating function but nonetheless are able to produce an infinite list.

This is probably a too long way of reconstructing my path through the last part of the paper, in which Greg discusses the Yablo's paradox using indexicals. He first proposes the generating function: “nEvery sentence below is untrue” , and argues that this isn't a generating function at all. I reply to this that it does not matter for the reasons above. He then reformulates the Yablo list as: “ All subsequent sentences are untrue”. At this point Greg claims that an utterance has a truth value at the time of which it is uttered not before, and it follows from this that at a given time we have just a finite list, and since a finite list is not paradoxical, we do not have any paradox. This reasoning, if it works, could also be applied to my proposal above: even if it would construct the infinite list, it would still fail to derive a paradox.
But I think Greg is too quick in granting that everyone would agree that a truth value of an utterance is gained at the moment that it is uttered and not before. I think this is not an uncontroversial view and need an argument: for sure a Platonist of some sort with respect of truth values would disagree entirely. Stretching it a little bit, it seems like the situation in which one says that the axiom of choice is neither true nor false until the moment one proves it one way or another. I am aware that this is a position that some people actually hold, but for sure it is not the only one: other people think that the axiom of choice is either true or false at any time, independently on what we can prove about it or when we can do it. Similarly, one could argue that no matter when or where a sentence is written down or uttered, it posses a truth value. In this case we have indexicals, so the situations could be trickier, in the sense that the truth value might depend on the context the sentence is in, but in my reading of Sn I am providing it, so it is unclear to me how Greg would respond to this. To conclude, I think that Greg's reply to the indexical version of the Yablo list is at the moment not very convincing, and probably needs more argumentation.

Therefore, one one hand I surely agree with Greg that Yablo fails in his attempt of producing a non-referntial version of the liar paradox, but I disagree about the motivations: while Greg thinks there is no paradox, I am inclined to think that the paradox is there, but it is self-referential.

Thanks!

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