What does Quantum Mechanics Tell us about Time?

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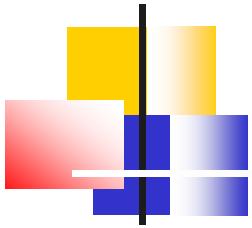
Outline

- Quantum Mechanics:
  - Orthodox QM
    - Time operator?
  - Heretical QM
    - Open future? Time quantization?
- Quantum Gravity:
  - Nothing is moving?
- String Theory:
  - No duration and length?
- Electrodynamics (if time permits ☺)
  - Chronon?

Classical Mechanics in a Nutshell



- Physical description:
  - The world is made of particles
  - *r(t)* represents the position of these particles in 3-d space
  - The solutions of Newton's equation (F=ma) represent possible physical states

Orthodox Quantum Mechanics (OQM) in a Nutshell

- Physical description
  - The world is made of wave function Ψ
    - Every physical object is describable in terms of the wave function
    - The solutions of the Schrödinger's equation represent possible physical states

Operators and Observables

- Eigenvalues and spectrum:
  - $A\Psi=\alpha\Psi$, α (the eigenvalue) is a complex number; the set of all eigenvalues is the spectrum of the operator
- Self-adjoint operator: real spectrum
- All observables physical quantities are associated to a given self-adjoint operator
- Eigenvalues provide the possible values that quantity can posses and that are revealed in an experiment

Operators and Observables

- Example: Energy
- Continuous spectrum
  - ex: free particle, scattering states
- Discrete spectrum
  - ex: bound states (Hydrogen atom)

Operators and Observables

- Important operators:
  - Position: $\hat{x}$ multiplies by x
  - Momentum $\hat{p}$ multiplies by $-i\hbar\frac{d}{dx}$
  - Angular momentum $\hat{L}_z$ multiplies by $-i\hbar\left(\frac{\partial}{\partial y}-\frac{\partial}{\partial z}\right)$
  - Energy $\hat{H}$ multiplies by $-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial r^2}+V$
- Quantization rules (how to get the self adjoint operator associated to observables with classical analog)

$$A(x,p,t) \rightarrow A(x,-i\hbar\frac{\partial}{\partial x},t)$$

Self-adjoint time operator?

- To every observable is associated a self adjoint operator
- What about time...?
  - We can measure time:
    - Time to go from here to there (Measurements of arrival times in scattering experiments)
    - Measurement of the half-life of radioactive nucleus
    - ...

Operators and Observables

- A particular feature of operators:
- Their product is not commutative
  - AB is not equal to BA!
- commutator [A,B]=AB-BA

Uncertainty Relations

- Position-momentum uncertainty:

$$\Delta x \Delta p \geq \hbar/2$$

  - position and momentum cannot be measured, at the same time definite, with precise values; if one has a precise value, say *x*, the other has to have an indefinite value of at least of $\Delta p = \hbar/2$.
  - It comes from the fact that the operators associated with x and p do not commute:

$$[\hat{x}, \hat{p}] = \hat{x}\hat{p} - \hat{p}\hat{x} = ih/2\pi$$

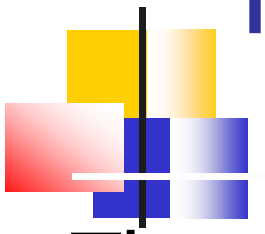
Uncertainty Relations

- Energy-time uncertainty:

$$\Delta E \Delta t \geq \hbar/2$$

- it is not obvious (and controversial) what this is supposed to mean since there is no time self-adjoint operator
  - Pauli's "theorem": if there were a time self-adjoint operator the spectrum of H would be the entire real line, forbidding discrete eigenvalues and making H unphysical since unbounded from below

The Problem of Time in OQM



- There is NO time operator!
  - Does that mean time is not measurable?
- That seems certainly wrong…so, what is going on?

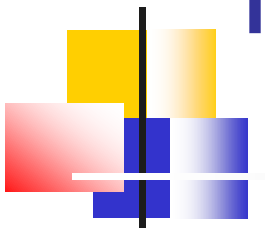
The Problem of Time in OQM

- Some resolutions:
  - Hilgevoord: it is just a product of the confusion between different notions of time
    - External time: like spatial coordinates, a parameter in the evolution, measured by clocks
      - → no s-a operator, no t-E uncertainty relation
    - Internal time: a dynamical variable specific to a given physical system (like the position of the tip of the hand of the clock relative to the background)
      - There can be a s-a operator, and correspondingly a t-E uncertainty relation

The Problem of Time in OQM

- Busch:
  - Observable time (aka event time): the time difference between events, the duration of a process (e.g. a time of arrival, or a time of flight, or a time of decay)
    - There can be a s-a operator, and correspondingly a t-E uncertainty relation
- Pashby:
  - Hilgevoord is mistaken in underestimating the importance of event time and, accordingly, of the role of POVM in OQM

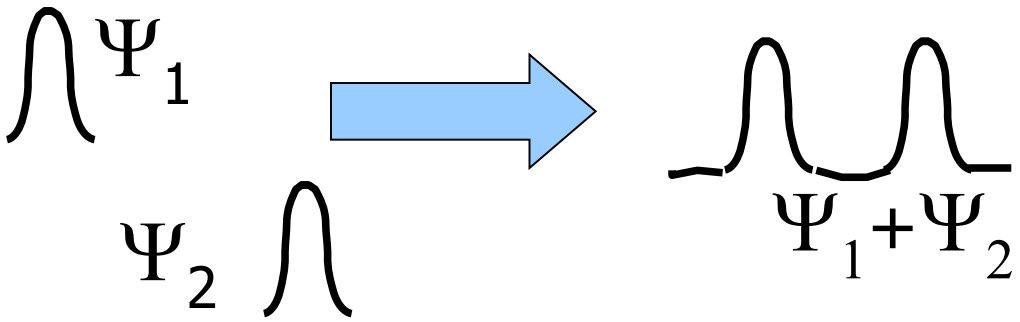
The Problem of Time in OQM



- Another possible answer:
  - Just reject the theory: we have already evidence that the theory is unsatisfactory (see later), here is another reason

The measurement problem and the failure of OQM

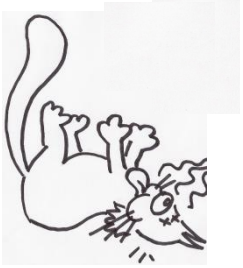
- S's equation is Linear: given Ψ_1 and Ψ_2 describe two possible physical states of a system at time t, also $\Psi_1+\Psi_2$ describe a possible physical state of the system at time t
  - State: everything that needs to be specified in order to completely the system under exam



The measurement problem and the failure of OQM

- States like $\Psi_1+\Psi_2$ are called superposition states
- Such states propagate to macroscopic systems, where they represent a problem since $\Psi_1+\Psi_2$ represents the sum of two macroscopically distinct states of the physical state under consideration → contradiction
  - Ex: a particle being detected "over here" (Ψ_1) AND "over there" (Ψ_1)
  - Ex: a cat being dead AND alive

AND



The measurement problem and the failure of OQM

Moral of the story:

- 1. the wave function provides a complete description of physical systems
- 2. the wave function evolves according to Schrödinger's equation

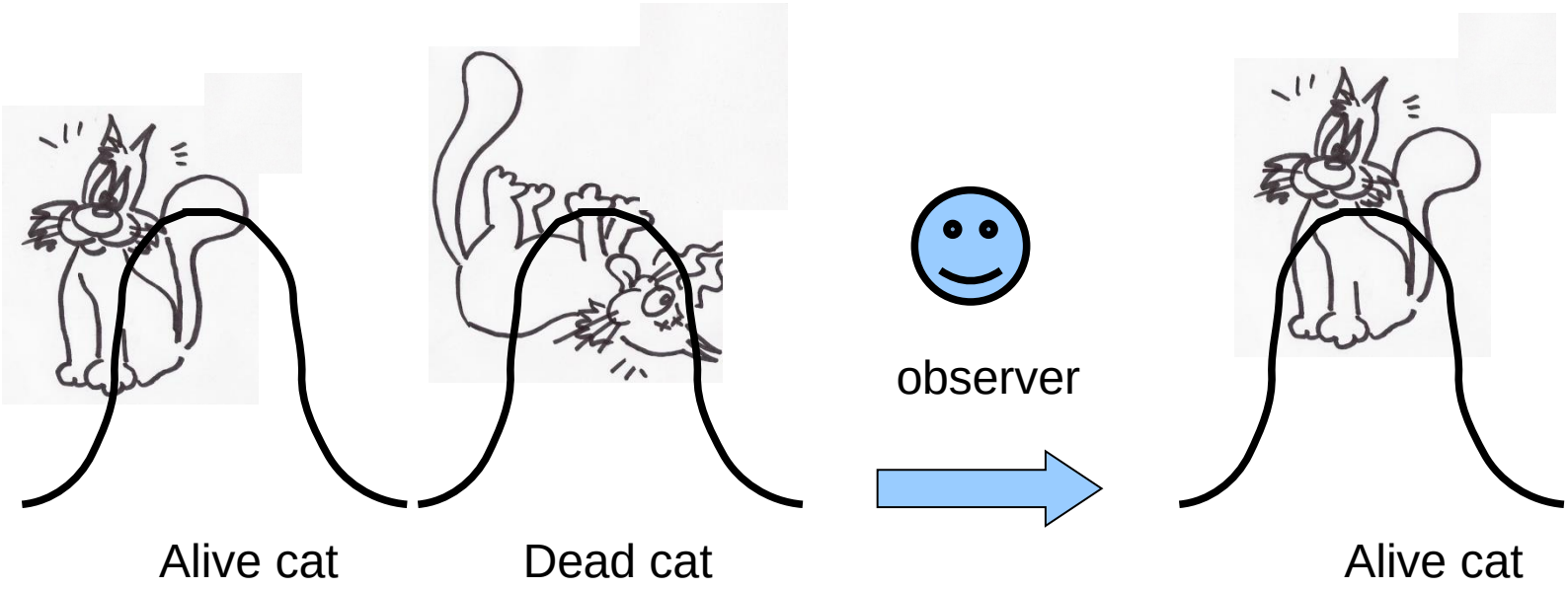
- Are incompatible (since they lead to macroscopic contradictory superpositions)

The solutions

- Deny 1 (the wave function is complete)
  - Add the observer (QM with the observer; von Neumann, Wigner)
  - Add macroscopic variables (Copenhagen interpretation; Bohr)
  - Add particles (Bohm's theory; de Broglie, Bohm)
- Deny 2 (the wave function evolves according to Schrödinger's equation)
  - The wave function evolves according to another equation (spontaneous localization theories; Ghirardi, Rimini, Weber, Pearl)

The Observer (von Neumann)

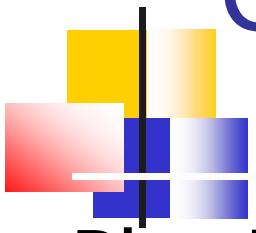
- When the observer looks (measures) a system the wave function changes its temporal evolution and "collapses" (randomly) into one of the terms of the superposition



Problems with this interpretation

- How to characterize the concept of observation? What makes a physical system an observer?
  - Is it only me, or all humans? Only humans or also animals? …
    - Intrinsically imprecise
- Does consciousness play any role in physics?
    - Intrinsically incomplete

Copenhagen Interpretation (Bohr)



- Physical systems are described by:
  - The wave function
    - → quantum world

    AND
  - Macroscopic variable (pointer positions)
    - → classical world

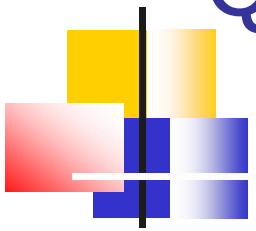
Problems with this interpretation

- These two worlds are incompatible with one another and are needed to supplement each other:
  - to have a complete theory of the world one need TWO theories: classical AND quantum mechanics
    - Intrinsically incomplete
- Problem of the cut: what is the line between a classical and a quantum object?
    - Intrinsically imprecise

Von Neumann and Bohr's interpretations

- Either one of them (it depends on the book you read) is referred as to OQM
- So everything we have said regarding time applies here

Quantum Theories without Observers

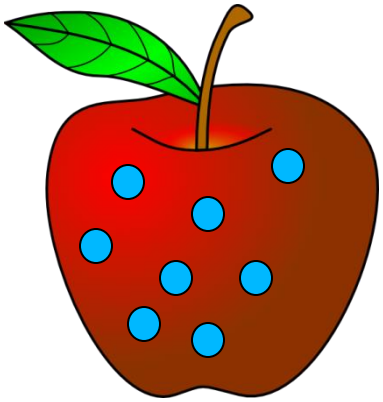


- All quantum theories (without the measurement problem) that do not need the specification of an observer to be complete:
  - Bohmian Mechanics
  - GRW theory
  - (Many-Worlds)

Bohmian Mechanics (de Broglie, Bohm)

- The world is made of particles evolving according to a law (the guiding equation) that involves the wave function

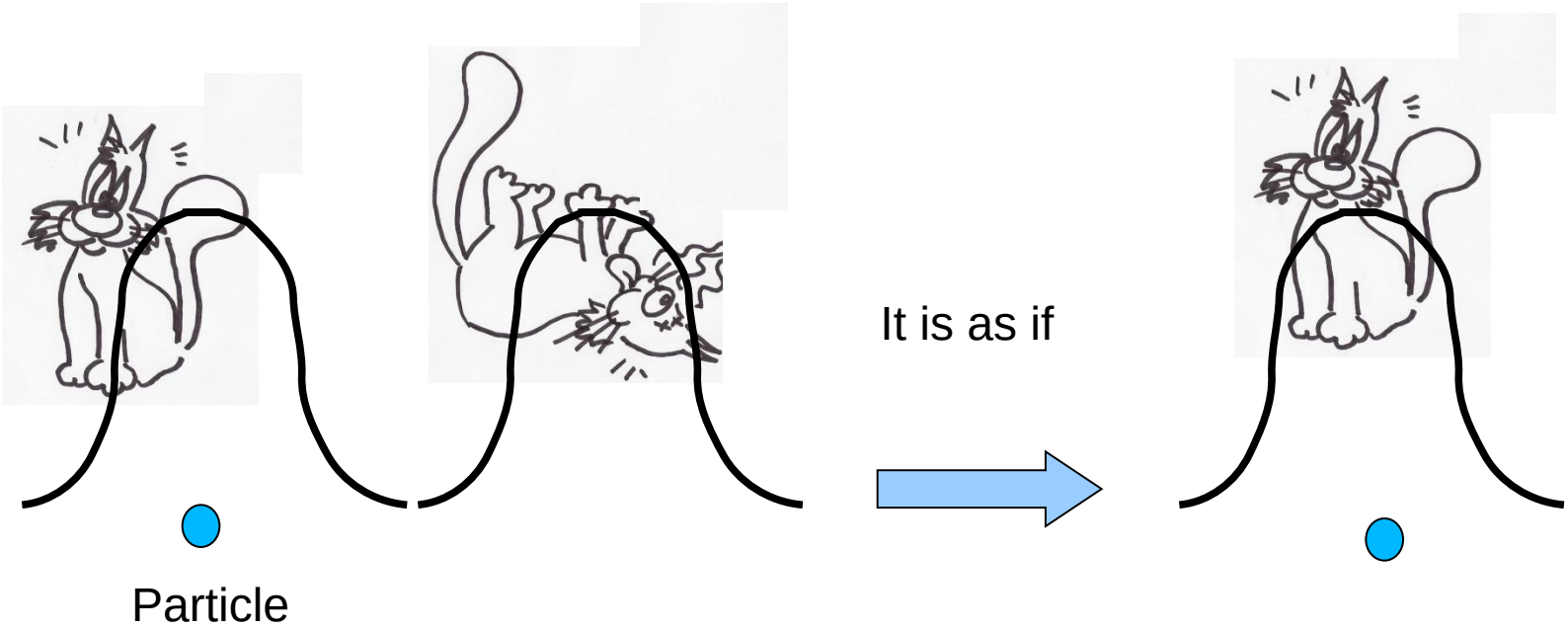
$$\frac{dQ_k}{dt} = \frac{\hbar}{m_k} Im \frac{\frac{\partial \psi}{\partial x}}{\psi}(Q_1, ..., Q_N)$$



- In turn, the wave function moves in time according to Schrödinger's equation

Bohmian Mechanics (de Broglie, Bohm)

- There are no macroscopic superpositions:
  - the particle's position (being "here" or "there") "decides" which term of the superposition actually happens



Bohmian Mechanics (de Broglie, Bohm)

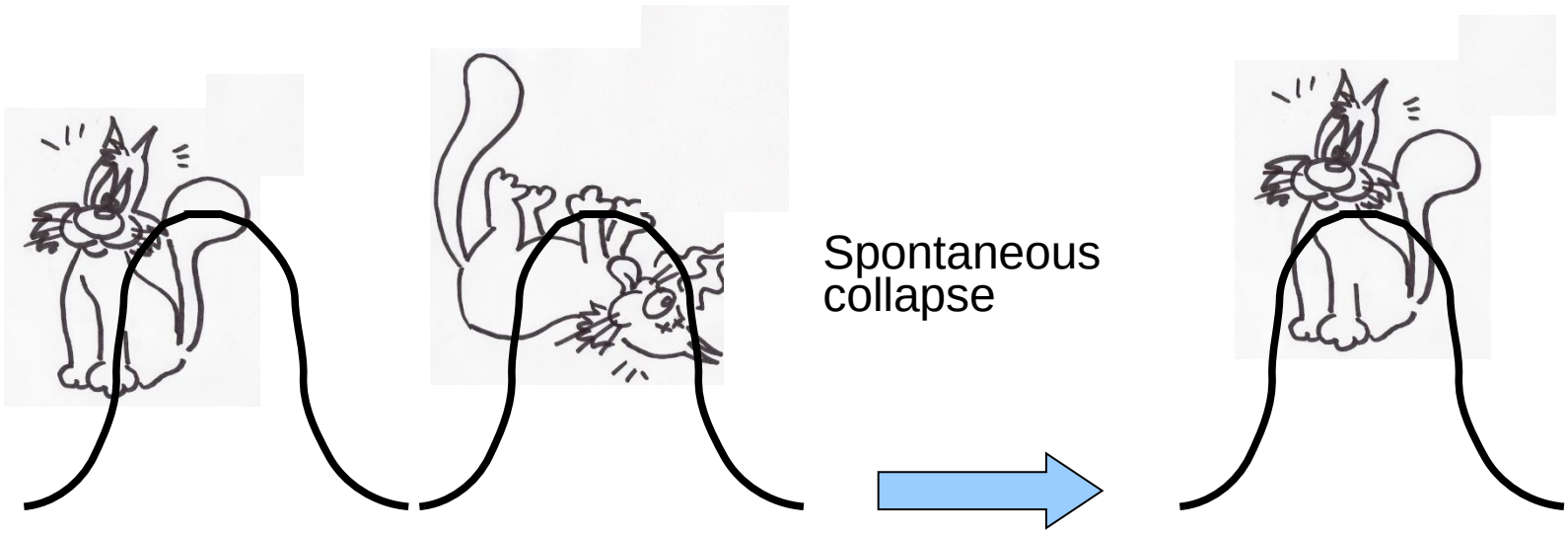
- In BM time plays the same role it plays in Classical Mechanics:
- a parameter in the evolution equation
  - → nothing new here!
- In BM self-adjoint operators do not describe properties, they summarize experimental results
  - The fact there is no time operator does not have any particular meaning

Spontaneous Localization (Ghirardi-Rimini-Weber theory)

- The wave function evolves according to a stochastic (i.e. random) equation: it evolves according to the Schrödinger equation most of the time but "sometimes" it spontaneously collapses into one of the terms of the superposition

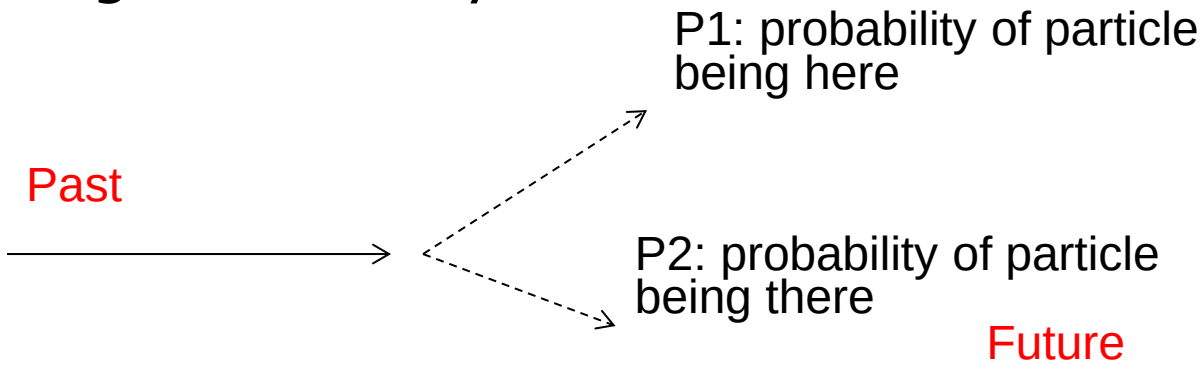
Spontaneous Localization (Ghirardi-Rimini-Weber theory)

- There are no macroscopic superpositions:
  - after a suitably short time (the shorter the more massive the object is) the wave function has spontaneously collapsed in one of the terms of the superposition



GRW and time

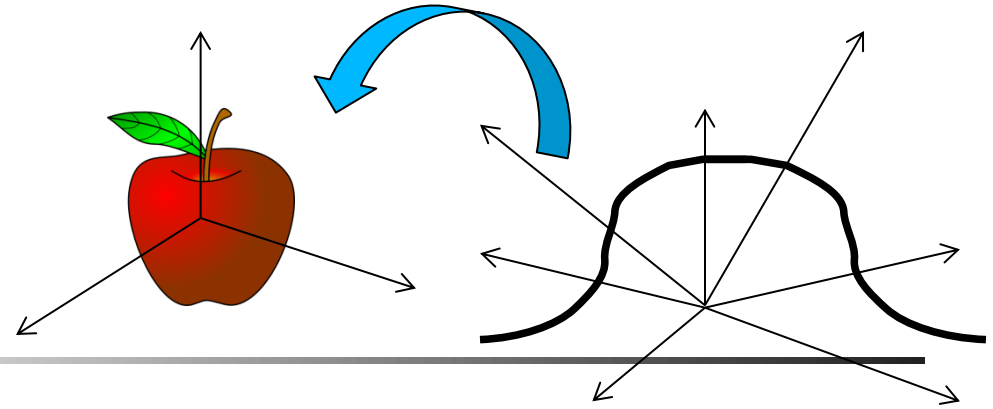
- It describes probabilities about the future, and not about the past:
  - Does that mean there is an intrinsic time directionality?
  - Does that mean that there is a fixed past and open future?
    - Growing block theory of time?



Spontaneous Localization (Ghirardi-Rimini-Weber theory)

- Controversial how to interpret the theory
- Three versions:
  - Bare (GRW0)- the world is made of wave functions in configuration space
  - Mass density (GRWm) - the world is made of a continuous mass density field (defined by the wave function) in 3-d space
  - Flashes (GRWf) - the world is made of events in space-time

GRW0



- The microscopic description of nature is in terms of a continuous matter field described by the wave function
  - NOTE: The wave function in NOT an object on 3-d space but on configuration space:

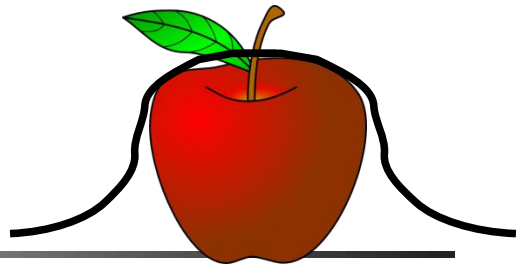
$$\psi(t) = \psi(r(t)_1, r_2(t), \ldots, r_N(t)) =$$
$$\psi(x_1(t), y_1(t), z_1(t), x_2(t), y_2(t), z_2(t),$$
$$\ldots, x_N(t), y_N(t), z_N(t))$$

→ Problem of emergence of 3-d space

GRW0 and time

- There seem to be no other additional insight on time:
  - it is (again) a parameter to obtain the evolution of the wave function

GRWm



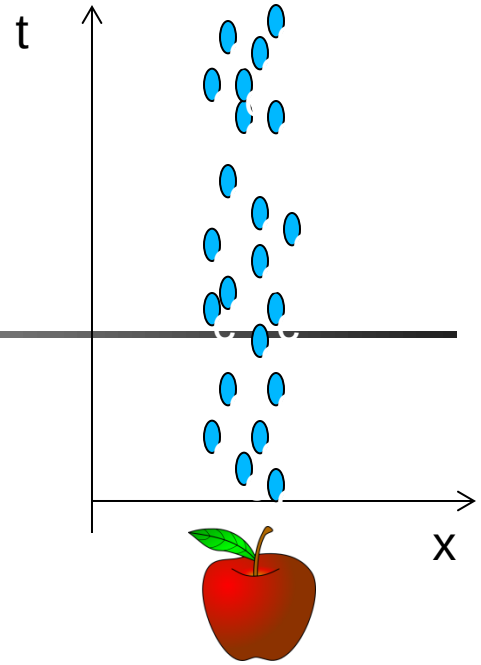
- The microscopic description of nature is in terms of a continuous matter field in 3-d space (that one can compute given the wave function)

$$m(x,t) = \sum_{i=1}^{N} m_i \int_{\mathbb{R}^{3N}} dq_1 \cdots dq_N \, \delta(q_i - x) \left|\psi(q_1, \ldots, q_N, t)\right|^2$$

GRWm and time

- There seem to be no other additional insight on time:
  - it is (again) a parameter to obtain the evolution of the wave function

GRWf



- It is a theory about events in space-time called flashes, whose position in s-t can be read off from the history of the wave function
- The microscopic description of nature is discrete in space-time!

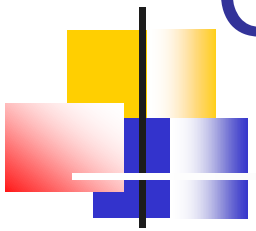
GRWf and time

- As in a discrete spatial picture not all space-points are occupied by matter, here also not all time-points are actually occupied by matter
- Open questions:
  - Is time (and space) intrinsically discrete?
  - Or is s-t continuous with a discrete matter in it?

Relativity and QM

- Attempts to reconcile QM and R:
  - Quantum Gravity
    - Apply the quantization rules to the equations of Relativity
  - String theory
    - Change QM using new fundamental objects (strings)
      - 11-d space, in which all dimensions are "compactified" but 3+1
      - graviton: mediator boson of the gravitational force

Canonical Quantum Gravity



- An approach to gravity quantization (Arnowitt-Deser-Misner)
- First, one has to put General Relativity "canonical" form
- Then apply the quantization rules

The problem of time in CQG

- Resulting equation (Wheeler-deWitt):

$$\widehat{H}\Psi = 0$$

- We have no time-dependent equation and therefore no time dependent wave function:
  - There is no evolution, no change (no time?), we only have a timeless wave function on configuration space

The problem of time in CQG

- Wheeler, Barbour and Rovelli: describe change without time relating physical systems directly to one another
- Earman: time is not real in CQG
- Maudlin: this should not be taken metaphysically seriously since it is all a consequence of forcing relativity into canonical form
- Healey:one can have a temporal evolution in GR, and attempts to explains how time can emerge in a fundamentally timeless reality

The problem of time in CQG

- Goldstein and Teufel:
  - CQG is conceptually flawed and thus it is dangerous to draw metaphysical lessons from it
  - The problem of time will disappear once we will consider a Bohmian quantum gravity in which the Wheeler-de Witt equation is supplemented by a guiding equation that provides for temporal development.

String Theory

- The world is made of oscillating strings (2-d objects)
- Oscillation modes are identified with the different particles (including the graviton)
- Weyl symmetry:
  - the length assigned to any curve has NO physical significance since it can be rescaled to anything one chooses
- Witten: emergence of space-time in string theory ?

String Theory

- Time in relativity depends on the frame of reference but it is still true that a time-like curve has a determinate duration (proper time)
- Not so in string theory:
  - the duration of any time-like curve and the length of a space-like curve ca be transformed to any value by a Weyl transformation

→ duration and length are not physically meaningful?

Electrodynamics

- There are many theories of the electron, all of which are very unsatisfactory(either infinities or unphysical solutions)
  - Abraham (spherical electron); Lorentz (Lorentz-Fitzgerald contraction); Poincare' (not only a pure electromagnetic mass); Dirac (point-like electron); Wheeler-Feynman (absorber theory)…

Electrodynamics

- Caldirola: theory of electron with discrete time - even if time flows continuously, there is a finite interval of proper time τ_0
  - When a force acts on the electron, the particles does not move continuously but the velocity "jumps" from $u(\tau-\tau_0)$ to $u(\tau)$
  - Dirac's equation is replaced by two finite differences equations

Electrodynamics

$$\frac{\tau_0}{2} \equiv \theta_0 = \frac{2}{3}\frac{ke^2}{m_0 c^3} \simeq 6.266 \times 10^{-24}\ \mathrm{s}$$

- for the electron= chronon
- The electron is an extended structure that behaves like a point-particle only in given positions
- Quantum extension: New finite differences discrete Schrödinger equation

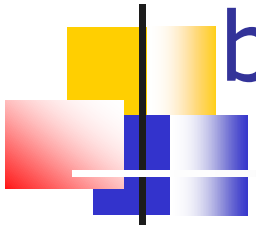
General considerations on the relation between physics and metaphysics

- Physics suggests, never forces, a metaphysics
  - There are always empirically equivalent but different possible mathematical formulations (underdetermination of theories from experimental data)
    - E.g.:
    - Classical mechanics: Newton, Hamilton, Lagrange formulations
    - Quantum mechanics: Copenhagen interpretation, Bohm theory,...
    - Special Relativity and Lorentz theory (length contraction)
    - General Relativity: "normal" formulation, canonical formulation,...

General considerations on the relation between physics and metaphysics

- The choice of the theory cannot be dictated by experiments but has to come from other (superempirical) considerations:
  - Explanatory power, elegance, simplicity, compatibility with other theories or metaphysics…

General considerations on the relation between physics and metaphysics



- E.g.: Orthodox Quantum Mechanics and Bohmian Mechanics
  - Are empirically equivalent
  - But BM is, arguably, more satisfactory than OQM
    - Clear relationship between physics and metaphysics
- E.g.: Lorentz and Einstein theory
  - Are empirically equivalent
  - But Lorentz postulates an additional force and is mathematically less elegant

General considerations on the relation between physics and metaphysics

- E.g.: General Relativity
  - The most natural formulation, mathematically, is the one in terms of space-time
  - It suggests that the flow of time is not real
  - But if there are other considerations that lead to believe that time is real, this is not necessarily in contrast with the theory