What does Quantum Mechanics Tell us about Time?

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Outline

- Quantum Mechanics:
  - Orthodox QM
    - Time operator?
  - Heretical QM
    - Time quantization? (GRWm)
- Electrodynamics:
  - Chronon?
- Quantum Gravity:
  - Nothing is moving?

Classical Mechanics in a Nutshell

- Mathematical description:
  - Fundamental object: $r = r(t) = (x(t), y(t), z(t))$
  - Fundamental equation: $m\frac{d^2 r(t)}{dt^2} = F$
  - Laws of the force
    - Electrical, gravitational,... $F = -\frac{d}{dr}V(r)$

Classical Mechanics in a Nutshell

- Physical description:
  - The world is made of particles
  - *r(t)* represents the position of these particles in 3-d space
  - The solutions of Newton's equation represent possible physical states
  - Every physical object has to be represented in terms of *r(t)*
    - Otherwise the theory is incomplete

Classical Mechanics in a Nutshell

- The position of particles completely describes every physical system

r ⟶ ● elettrone

$$(r_1, r_2, .., r_N); r_{CM} = \frac{m_1 r_1 + m_2 r_2 + ... + m_N r_N}{m_1 + m_2 + ... + m_N}$$ ⟶

- A system composed of N particles is represented by N-ples of the positions of the particles composing it or by its center of mass

(Orthodox) Quantum Mechanics in a Nutshell

- A fundamental physical theory
  - Mathematical description
    - Fundamental object: $\psi = \psi(r_1, r_2, .., r_N, t)$ (N~10^{23})
    - Fundamental equation: $i\hbar\frac{\partial\psi}{\partial t} = H\psi$
    - Hamiltonian (in which there are potentials)
      - Electrical, gravitational,...

$$H = -\frac{\hbar^2}{2\text{m}}\frac{d^2}{dr^2} + V(r_1, \ldots, r_N)$$

(Orthodox) Quantum Mechanics in a Nutshell

- Physical description
  - The world is made of wave function
    - Every physical object has to be describable in terms of the wave function
    - The solutions of the Schrödinger equation represent possible physical states
  - *Trying to give sense to these claims has given rise to the various interpretations of QM*

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The wave function and the position

- The position is a function in 3-d space:

$$r(t) = (x(t),\ y(t),\ z(t))$$

- The wave function is a function on a much larger space:

$$\psi(t) = \psi(r(t)_1, r_2(t), \ldots, r_N(t)) =$$
$$\psi(x_1(t), y_1(t), z_1(t), x_2(t), y_2(t), z_2(t),$$
$$\ldots, x_N(t), y_N(t), z_N(t))$$

That is, a space with $3N \sim 10^{23}$ dimensions

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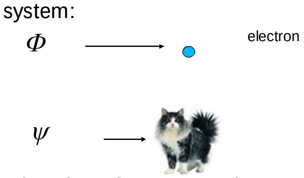
Quantum Mechanics in a Nutshell

- The wave function completely describes every physical system:



- The wave function of a composed system is given by the product of the wave functions of its components

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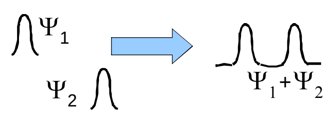
Quantum Mechanics in a Nutshell

- S's equation is Linear: given Ψ_1 and Ψ_2 describe two possible physical states of a system at time t, also $\Psi_1+\Psi_2$ describe a possible physical state of the system at time t
  - State: everything that needs to be specified in order to completely the system under exam



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The problem of the S's cat

Experiment:

- There is a cat in a box together with a device activated by the decay of a radioactive nucleus
- If the nucleus does not decay, nothing happens and the cat is alive
- If the nucleus decays, the device activates, deliver the poison in the box and the cat dies

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The problem of the S's cat

- At time t_0 the possible initial states (solutions of S's equation) for the nucleus are:

$$\psi_{t0,decayed}\ ,\ \psi_{t0,Not\ decayed}$$

- At time t_0 the cat's initial state is:

$$\Phi_{t0,alive}$$

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The problem of the S's cat

- Since the equation is linear, another possible initial state (at time t_0) for the nucleus is:

$$\psi_{t0,decayed} + \psi_{t0,Not\ decayed}$$

- The corresponding final state (at the final time t') of the composite system (cat+nucleus) is:

$$\Psi_{t'} = \psi_{t',decayed}\,\Phi_{t',dead} + \psi_{t',Not\ decayed}\,\Phi_{t',alive}$$

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The problem of the S's cat

- What is the meaning of this state?

$$\psi_{t',decayed}\,\Phi_{t',dead} + \psi_{t',Not\ decayed}\,\Phi_{t',alive}$$

- It is the sum of the state describing a decayed nucleus and a dead cat, AND an undecayed nucleus and a live cat
- Superposition state: it is the sum of two macroscopically distinct states of the physical state under consideration
  - ????????!!!!!!!!

 AND 

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The problem of the S's cat

- Fro our experience we know that macro systems are NEVER in a "superposition"
  - When we look into the box we either see a dead cat or an alive one!

 OR 

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The problem of the S's cat

- But from what we have seen, IF the wave function provides a complete description of physical systems AND evolves according to the Schrödinger equation THEN produces superposition states all the time
- Thus, if we want QM to be able to describe what actually happens we have two choices (Bell, 1987):
  - Either the wave function does not provide the complete description of physical systems
  - Or it does not evolve according to S's equation

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The problem of the S's cat

Moral of the story:

- 1. the wave function provides a compekte description of physical systems
- 2. the wave function evolves according to S's equation

- Are incompatible

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The solutions

- Deny 1 (the wv if complete)
  - Add the observer (QM with the observer; von Neumann, Wigner)
  - Add macroscopic variables (Copenhagen interpretation; Bohr)
  - Add particles (Bohm's theory; de Broglie, Bohm)
- Deny 2 (the wv evolves according to S's equation)
  - The wv evolves according to another equation (spontaneous localization theories; Ghirardi, Rimini e Weber, Pearl)

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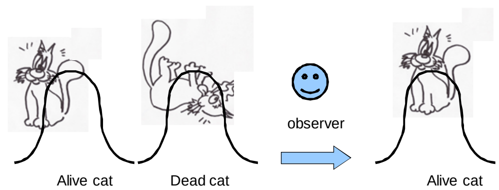
The observer

- When the observer looks (measures) a system, the wv changes its temporal evolution and "collapses" (randomly) into one of the terms of the superposition



Problems with this interpretation

- How to characterize the concept of observation?
- What makes a system an observer?
  - Is it only me, or all humans? Only humans or also animals? ...
- Does consciousness play any role in physics?
- Can we do without the observer?
  - Yes: in all other interpretations....

Copenhagen Interpretation

- Physical systems are described by:
  - The wave function
  - Macroscopic variable-experiments:
    - Experiments are made to measure physical quantities (position, momentum, energy, ...)
    - To each experiment is associated a self adjoint operator
    - The possible results of experiment A are the eigenvalues of the operator $\hat{A}$ associated to A

Copenhagen Interpretation

- Operators acts on wave functions
  - es: $A\,\Psi = \Psi'$
- Linear: $A(\alpha\Psi+\beta\Phi)=\alpha A\Psi+\beta A\Phi$
- Eigenvalues, eigenvectors and spectrum:
  - $A\Psi=\alpha\Psi$, α is a complex number
- Adjoint of A: $A^{+}\Psi=\alpha^{*}\Psi$
- Self-adjoint operator: $A^{+}=A$, real spectrum

Copenhagen Interpretation

- All observables physical quantities are associated to a given self-adjoint operator
- Eigenvalues provide the possible values that quantity van posses

Copenhagen Interpretation

- Energy:
- Continuous spectrum
  - ex: free particle, scattering states
- Discrete spectrum
  - ex: bound states (Hydrogen atom)
  - Quantization of momentum, of angular momentum, of energy
    - (the only case in which one associates a particle to a quantization is the one of energy, with the pohoton)

Copenhagen Interpretation

- Important operators:
  - Position: $\hat{x}$ multiplies by *x*
  - Momentum $\hat{p}$ multiplies by $-i\hbar\frac{d}{dx}$
  - Angular momentum $\hat{L}_z$ multiplies by $-i\hbar\left(\frac{\partial}{\partial y}-\frac{\partial}{\partial z}\right)$
  - Energy $\hat{H}$ multiplies by $-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial r^2}+V$
- Quantization rules (how to get the self adjoint operator associated to observables with classical analogue)

$$A(x,p,t) \rightarrow A(x,-i\hbar\frac{\partial}{\partial x},t)$$

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Copenhagen Interpretation

- To every observable is associated a self adjoint operator
- What about time...?
  - We can measure time:
    - Time to go from here to there (Measurements of arrival times in scattering experiments)
    - Measurement of the half-life of radioactive nucleus
    - ...

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Copenhagen Interpretation

- There is NO time operator!
  - SO TIME IS NOT MEASURABLE!
- What is going on?
- Should we infer from this that the physical quantity “time” has a special status?

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Copenhagen Interpretation

- The t variable associated to time appears uin the equations of QM as it appears in the equations of CM:
  - Objects evolve in time
    - There is space-time separation, not as in Relativity

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Copenhagen Interpretation

- A possible answer:
  - Just reject the theory - this is a (further) claim it is flawed

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Copenhagen Interpretation

- Another possible answer:
  - All measurements are position measurements
    - E.g.: Arrival times results are measured by the position if the pointer of the clock
- If so, though, the only necessary operator is the position operator!

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Copenhagen Interpretation

- A particular feature of operators:
- Their product is not commutative
  - AB is not equal to BA!
- commutator [A,B]=AB-BA

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Copenhagen Interpretation

- Position-momentum uncertainty:

$$\Delta x \Delta p \geq \hbar/2$$

  - If one measures the position with absolute precision, the momentum will be uncertain of a given amount and vice versa
- It comes from the fact that the operators associated to position and momentum do not commute:
  - $[\hat{x},\hat{p}] = \hat{x}\hat{p} - \hat{p}\hat{x} = ih/2\pi$

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Copenhagen Interpretation

- Energy-time uncertainty:

$$\Delta E \Delta t \geq \hbar/2$$

- it is not obvious (and controversial) what this is supposed to mean since there is no time operator....

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Other Interpretations

- Bohmian Mechanics
  - The world is made of particles evolving according to a law (the guiding equation) that involves the wave function, which in turn evolves according to S's equation:

$$\frac{dQ_k}{dt} = \frac{\hbar}{m_k} Im \frac{\frac{\partial \psi}{\partial x}}{\psi}(Q_1, ..., Q_N)$$

  - Time has the same role here as it has in CM

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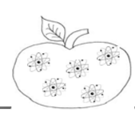
Metaphysics of BM



- The microscopic description of nature is in terms of particles (independent of the wave function)
- The only property is position
- Self-adjoint operators do not describe properties, they summarize experimental results
  - The fact there is no time operator does not have any destructive meaning

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Other interpretations

- Spontaneous Localization (Ghirardi-Rimini-Weber theory)
  - Two versions:
  - Mass density (GRWm)

$$m(x,t) = \sum_{i=1}^{N} m_i \int_{\mathbb{R}^{3N}} dq_1 \cdots dq_N \, \delta(q_i - x) \left|\psi(q_1, \ldots, q_N, t)\right|^2$$

  - flashes (GRWf)

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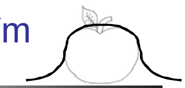
Metaphysics of GRWm



- The microscopic description of nature is in terms of a continuous matter field (that one can compute given the wave function)
- Time is (again) a parameter to obtain the evolution of the wave function and the mass density
  - No novelty

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Metaphysics of GRWf

- It is a theory about events in space-time called flashes, whose position in s-t can be read off from the history of the wave function
- The microscopic description of nature is discrete in space-time!

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Electrodynamics

- There are many theories of the electron, all of which are very unsatisfactory
  - Abraham (1904): theory of the spherical electron
    - Pure electromagnetic mass, uniformly distributed on the surface
    - problem: incompatible with Relativity
  - Lorentz (1904): Lorentz-Fitzgerald contraction
  - Poincare' (1906): new relativistic theory assuming not only a pure electromagnetic mass
    - Problem: divergences

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Electrodynamics

- Dirac (1928): point-like electron theory+ tricks to avoid infinities
  - Problem: many unphysical solutions
- Wheeler-Feynman: absorber theory
  - compatible with Relativity but with other problems when trying to make this theory compatible with QM (QED)
  - When taking into account the self-field of the electron the theory "diverges"

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Electrodynamics

- Caldirola: classical theory of electron with discrete time
- Even if time flows continuously, there is a finite interval of proper time τ_0

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Electrodynamics

- When a force acts on the electron, the particles does not move continuously but the velocity "jumps" from $u(\tau-\tau_0)$ to $u(\tau)$ along the trajectories of the particles in s-t

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Electrodynamics

- Dirac's equation is replaced by two finite differences equations:
  - Equation for velocity (reduces to D's eq when τ_0 goes to 0)

$$\frac{m_0}{\tau_0}\left\{u_\mu(\tau) - u_\mu(\tau-\tau_0) + \frac{u_\mu(\tau)\,u_\nu(\tau)}{c^2}\left[u_\nu(\tau) - u_\nu(\tau-\tau_0)\right]\right\} = \\ = \frac{e}{c}F_{\mu\nu}(\tau)\,u_\nu(\tau),$$

  - Transition law between two positions

$$x_\mu(n\tau_0) - x_\mu[(n-1)\tau_0] = \frac{\tau_0}{2}\left\{u_\mu(n\tau_0) - u_\mu[(n-1)\tau_0]\right\},$$

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Electrodynamics

$$\frac{\tau_0}{2} \equiv \theta_0 = \frac{2}{3}\frac{ke^2}{m_0c^3} \simeq 6.266 \times 10^{-24}\ \mathrm{s}$$

- for the electron= chronon
- The electron is an extended structure that behaves like a point-particle only in given positions

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Electrodynamics

- Quantum extension:
- New finite differences discrete Schrödinger equation

$$i\frac{\hbar}{\tau}\left[\Psi(\mathbf{x},\mathbf{t}) - \Psi(\mathbf{x},\mathbf{t}-\tau)\right] = \hat{H}\Psi(\mathbf{x},\mathbf{t}),$$

$$i\frac{\hbar}{2\tau}\left[\Psi(\mathbf{x},\mathbf{t}+\tau) - \Psi(\mathbf{x},\mathbf{t}-\tau)\right] = \hat{H}\Psi(\mathbf{x},\mathbf{t}),$$

$$i\frac{\hbar}{\tau}\left[\Psi(\mathbf{x},\mathbf{t}+\tau) - \Psi(\mathbf{x},\mathbf{t})\right] = \hat{H}\Psi(\mathbf{x},\mathbf{t}),$$

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Electrodynamics

- Consequences:
  - The muon and the tau are not independent particles but can be identified with excited states of the electron

$$m^{(n,p)} = \left[1+(2p)^4\right]m^{(n)} = m_0\left[1+(2p)^4\right]\left[\frac{3}{2}\frac{1}{\alpha}+1\right]^n.$$

n	p	$m^{(n)}$	
0	0	0.511 MeV	electron
1	0	105.55 MeV	muon
	1	1794.33 MeV	tau

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Electrodynamics

- Consequences for the nature of time:
  - Time flows continuously
  - There is a fundamental unit of proper time (one for each fundamental particle)
  - This is not a discretization of time itself
  - Rather it is a discretization of the evolution of a system along its trajectory in space-time
    - More investigation needed (under-studied theory)

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Relativity and QM

- Attempts to reconcile QM and R:
  - Quantum Gravity
    - Apply the quantization rules to the equations of Relativity
  - String theory
    - Change QM using new fundamental objects (strings)
      - 11-d space, in which all dimensions are "compactified" but 3+1
      - graviton: mediator boson of the gravitational force

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Canonical Quantum Gravity

- An approach to gravity quantization (Arnowitt-Deser-Misner)
- First, one has to put General Relativity "canonical" form
- Then apply the quantization rules

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Canonical Quantum Gravity

- Canonical form
  - Lagrangia: $L=K-V$,
  - Two conjugate variables q,p: $p = \frac{\partial L}{\partial q}$
  - Hamiltonian: $H = p\dot{q} - L$
  - Hamilton's equations:

$$\dot{q} = \frac{dq}{dt} = \frac{\partial H}{\partial p} \qquad \dot{p} = \frac{dp}{dt} = -\frac{\partial H}{\partial q}$$

  - Trajectories in space (q,p): physical history of the system

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Canonical Quantum Gravity

- Quantization:
  - Substitution of the q,p variables with the corresponding operators

$$H(q,p) \to H(q, -i\frac{\partial}{\partial q})$$

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Canonical Quantum Gravity

- General Relativity: Choice of the canonical variables
  - 3-metric $g_{ij}(x^a)$ as q
  - Extrinsic curvatue $\pi^{ij}(x^a)$ as p
- Hamilton equation:

$$\begin{aligned} dg_{ij}(x^a) &= \frac{\delta H_N(g,\pi)}{\delta \pi^{ij}(x^a)} d\tau \\ d\pi^{ij}(x^a) &= -\frac{\delta H_N(g,\pi)}{\delta g_{ij}(x^a)} d\tau \end{aligned}$$

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Canonical Quantum Gravity

- Quantization: $g_{ij} \to \hat{g_{ij}}$

$$\pi_{ij} \to \hat{\pi_{ij}} = -i\frac{\delta}{\delta g_{ij}}$$

- Resulting equation (Wheeler-deWitt)

$$\widehat{H}\Psi = 0$$

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Canonical Quantum Gravity

- IN OQM we have a time-dependent wave function which provides the probability distribution of the measurement results of a given observable
- Here we have no time-dependent equation and therefore no time dependent wave function
- There is no evolution, we only have a timeless wave function on configuration space

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Canonical Quantum Gravity

- The problem of time:
- The WdW equation is an equation in configuration space, not in space-time, and nothing evolves
  - How is it possible that it successfully describes a universe like ours in which, everything changes???
  - Where the heck is time????

Canonical Quantum Gravity

- In “classical” Relativity
  - Newtonian time is gone, but the notion of time, for an observer somewhere in space-time who uses some given reference frame, “emerges” from space-time
- In Canonical Quantum Gravity
  - Space-time disappears and there is only configuration space

Canonical Quantum Gravity

- What does that mean?
- Earman-Maudlin dispute:
  - Earman says that this means that time is not real in CQG
    - It is all an illusion
  - Maudlin says that this should not be taken metaphysically seriously
    - It is all a consequence of forcing relativity into canonical form

Canonical Quantum Gravity

- E.g.: classical electrodynamics
  - The electric and magnetic fields can be written in terms of the vector and the scalar potentials:

$$E = -\frac{\partial \phi}{\partial x} - \frac{\partial A}{\partial t}$$

$$B = rot A$$

  - Different A and ϕ can provide the same E and B, wh
  - Fixing the gauge one can make them unique

Canonical Quantum Gravity

- E and B have physical significance, potentials do not
- If one insists to write the evolution equations in terms of the potentials she will obtain is indeterministic equations (the state at time t does not determine the state at subsequent times)
- But this indeterminism is not physical, it is only due to the freedom of choosing a gauge

Canonical Quantum Gravity

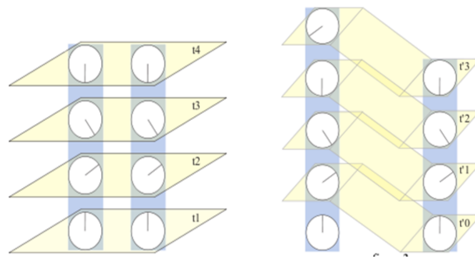
- Possibile solutions if one is willing to return to a deterministic dynamics:
  - Fix the gauge (specify an additional condition)
  - Quotient: divide in equivalence classes (gauge orbits)
    - States in the same class have the same physical significance

Canonical Quantum Gravity

- The case of General Relativity:
- If one insist in writing it in canonical form one needs the state at an instantaneous time while in GR there is no global simultaneity
- One needs to “slice” space-time
  - Each slice represents the universe at a given instant
- There is freedom of slicing as one likes

Canonical Quantum Gravity

Foliation 1 Foliation 2

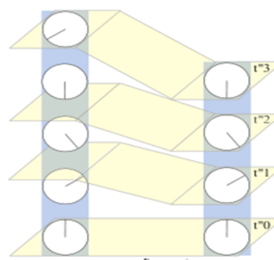


Canonical Quantum Gravity

- Given the foliation, one writes the Hamiltonian H, quantizes it, and obtains the solutions

Canonical Quantum Gravity

Foliation 3



Canonical Quantum Gravity

- The three foliations correspond to different trajectories: F1 will be on ein which the clocks all indicate the same ti,e F2 one in which the clocks indicate all a different time, F3 starts like F1 and then morph into F2.

Canonical Quantum Gravity

- we have indeterminism: the foliation at a given time does not determine the foliation at a subsequent time and thus the possible “future” solutions are not determined by the “past” ones
- But this is completely unphysical, it is simply due to the possibility of arbitrarily choose the foliation

Canonical Quantum Gravity

- Solution: equivalent classes?
- **NO** because:
  - 3 and 1 would be in the same class and thus would represent the same physics
    - This is absurd since the clocks tell a different time
- Answer: they seem different but they are actually the same
  - This is implausible since there does not seem to be another reason to maintain they are the same

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Canonical Quantum Gravity

- Goldstein and Teufel (2001):
- Canonical quantum gravity is conceptually flawed and thus it is dangerous to draw metaphysical lessons from it
- The problem of time will disappear once we will consider a Bohmian quantum gravity
  - The basic variable is the 3-metric, whose change under deformations is given by a vector field on configuration space generated by the wave function (solution of the E=WdW equation)

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General considerations on the relation between physics and metaphysics

- Physics suggests, never forces, a metaphysics
  - There are always empirically equivalent but different possible mathematical formulations (underdetermination of theories from experimental data)
    - E.g.:
    - Classical mechanics: Newton, Hamilton, Lagrange formulations
    - Quantum mechanics: Copenhagen interpretation, Bohm theory,...
    - Special Relativity and Lorentz theory (length contraction)
    - General Relativity: "normal" formulation, canonical formulation,...

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General considerations on the relation between physics and metaphysics

- The choice of the theory cannot be dictated by experiments but has to come from other (superempirical) considerations:
  - Explanatory power, elegance, simplicity, compatibility with other theories or metaphysics...

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General considerations on the relation between physics and metaphysics

- E.g.: Orthodox Quantum Mechanics and Bohmian Mechanics
  - Are empirically equivalent
  - But BM is, arguably, more satisfactory than OQM
    - Clear relationship between physics and metaphysics

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General considerations on the relation between physics and metaphysics

- E.g.: Lorentz and Einstein theory
  - Are empirically equivalent
  - But Lorentz postulates an additional force and is mathematically less elegant

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General considerations on the relation between physics and metaphysics

- E.g.: General Relativity
  - The most natural formulation, mathematically, is the one in terms of space-time
  - It suggests that the flow of time is not real
  - But if there are other considerations that lead to believe that time is real, this is not necessarily in contrast with the theory

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