On the Common Structure of Bohmian Mechanics and the Ghirardi-Rimini-Weber Theory

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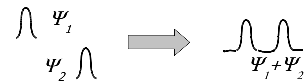
What is Quantum Mechanics?

- A fundamental physical theory
- The fundamental object of the theory is the wave function Ψ: it completely describes the state of a physical system
  - The wave function lives in configuration space (dimension $d \sim 10^{23}$)
  - The wave function evolves in time according to an equation called Schrödinger's equation

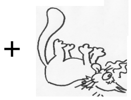
What is Quantum Mechanics?

- The equation is Linear: If Ψ_1 and Ψ_2 describe possible physical states at a given time t, also $\Psi_1+\Psi_2$ does
  - State: all you need to specify in order to completely describe the system



Impossible cats

- Because of linearity of the evolution equation, the wave function evolves into a superposition state:
- It is the sum of two macroscopically distinct states of affairs of the system under consideration (cat alive and cat dead)

 + 

Impossible cats

- From experience we know that macroscopic systems are NEVER in a superposition. Rather, they are always in well defined states

 or 

Impossible cats

- But we just saw that IF the wave function provides a complete description of a system AND it evolves according to Schrödinger's equation, THEN it produces such superpositions
- Therefore, IF we want quantum mechanics to describe what really happens (that is, if we want measurements to have results), THEN
- we have two choices (Bell, 1987):
  - Either the wave function does not provide the complete description
  - OR it does not evolve according to Schrödinger's equation

Impossible cats

Moral of the story:

- The three claims
  - 1:The wave function provides a complete description
  - 2:The wave function evolves according to Schrödinger's equation
  - 3:Measurements have results
- Are incompatible

Solutions to the measurement problem (without the observer)

- Deny claim 1 (the wave function provides a complete description)
  - Add particles positions (Bohmian Mechanics, BM)
- Deny claim 2 (the wave function evolves according to Schrödinger's equation)
  - The wave function evolves according to a stochastic equation (GRW theory)
- Deny claim 3 (measurements don't have results)
  - There is a multiverse of different worlds (Many Worlds, MW)

Bohmian mechanics and GRW

- BM and GRW would seem to have little in common:
  - They choose different horns of Bell's alternative
- The suggestion here is instead that BM and GRW theory have much more in common than one would expect at first sight:
  - They are mathematical structures grounded on a primitive ontology (PO)

Bohmian Mechanics

- Theory of particles in motion
- Complete description (Q,Ψ):
  - $Q=(Q_1, \dots, Q_N)$, Q_k in $\mathbf{R}^3$, k=1,...,N
  - $\Psi(Q)=\Psi(Q_1, \dots, Q_N)$

Bohmian Mechanics

•Guide equation

$$\frac{dQ_i}{dt} = v_i^\psi(Q_1,\dots,Q_N) = \frac{\hbar}{m_i}\mathrm{Im}\frac{\psi^*\nabla_i\psi}{\psi^*\psi}(Q_1\dots,Q_N)$$

•Schrödinger equation

$$i\hbar\frac{\partial\psi}{\partial t} = H\psi \qquad H = -\sum_{k=1}^{N}\frac{\hbar^2}{2m_k}\nabla_k^2 + V$$

Bohmian Mechanics

- $|\Psi(q)|^2$ *is equivariant:*
  - if the configuration $Q(t) = (Q_1(t), \dots, Q_N(t))$ is random with distribution $|\Psi_t(q)|^2$ at t, then this will be true also for any other time
  - If $\rho(q,t_0) = |\psi(q,t_0)|^2$

at some time t_0 then $\rho(q,t) = |\psi(q,t)|^2$

for all t

Bohmian Mechanics

- From the law of large numbers and the assumption that the initial configuration of the universe is typical for the $|\Psi_t|^2$ follows:
  - quantum equilibrium hypothesis (QEH): when the wave function of a system is Ψ_t, the configuration of the system $Q(t)$ is random with distribution $|\Psi_t|^2$
- As a consequence of QEH:
  - a Bohmian universe appears random
  - The predictions of BM will be exactly and always equal to the predictions of QM

GRW Theory

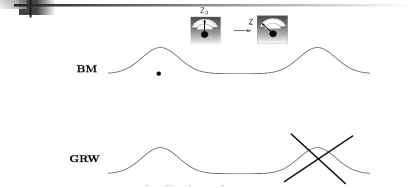
- $\Psi(Q)=\Psi(Q_1, \dots, Q_N)$
- $Q=(Q_1, \dots, Q_N)$, Q_k in $\mathbf{R}^3$, k=1,...,N
- "particles" are not really there

GRW Theory

- For any point x in $\mathbf{R}^3$ ⇒ $\Lambda_i(x) = \frac{1}{(2\pi\sigma^2)^{3/2}} e^{-\frac{(\hat{Q}_i - x)^2}{2\sigma^2}}$
- The evolution for ψ is Schrödinger interrupted by collapses ($\sigma \sim 10^{-7} m$)
- A collapse center with center x and label i will occur at rate ⇒ $r(x,i|\psi) = \lambda\langle\psi\,|\,\Lambda_i(x)\psi\rangle$ ($\lambda \sim 10^{-15}\, s^{-1}$)
- When this happens:

$$\psi \rightarrow \Lambda_i(x)^{1/2}\psi/\|\Lambda_i(x)^{1/2}\psi\|$$

Comparison between BM and GRW

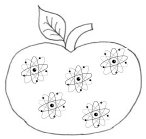


Localization ⇒ larger p ⇒ increase of energy (global warming)

GRW: DIFFERENT PREDICTIONS!

Bohmian metaphysics

- BM is about particles in 3-dimensional space :
  - The microscopic description of reality is discrete (particle-like)



GRW metaphysics?

- In GRW there is just the wave function.
- Is GRW a theory about the wave function?
- Problems of considering tables and chairs as made of wave functions:
  - The wave function lives in a space with a very large number of dimensions (~10^{23})
  - Where is three-dimensional space?

GRW metaphysics?

- "[...] the wave function as a whole lives in a much bigger space, of 3N dimensions. It makes no sense to ask for the amplitude or phase or whatever of the wave function at a point in ordinary space. It has neither amplitude nor phase nor anything else until a multitude of points in ordinary three-space are specified." [Bell, 1987]

Mass density GRW - GRWm

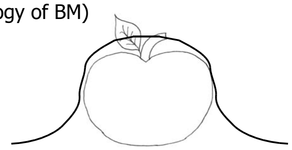
- GRWm is a theory about the behavior of a field *m(x, t)* on three-dimensional space

$$m(x,t) = \sum_{i=1}^{N} m_i \int_{\mathbb{R}^{3N}} dq_1 \cdots dq_N \, \delta(q_i - x) \left|\psi(q_1,\ldots,q_N,t)\right|^2$$

- This is reminiscent of Schrödinger's early view of the wave function as representing a continuous matter field.

GRWm metaphysics

- The microscopic description of reality provided by the matter density field *m(x, t)* is continuous (in contrast with the particle ontology of BM)



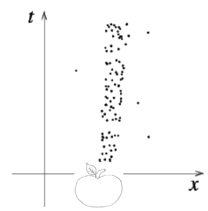
Flashy GRW - GRWf

- GRWf is a theory about a set of "events" in space-time, the flashes = the points in s-t corresponding to the collapses of the wave function
  - The wave function evolves in a random way
  - F is a random set of space-time

$$F = \{(X_1, T_1), \ldots, (X_k, T_k), \ldots\}$$

$N = 10^{23} \to 10^{8}$ flashes/second

GRWf metaphysics



- The microscopic description of reality provided by GRWf is discrete in space-time
- "the world is a galaxy of such events" [Bell 1976]

The need for a clear ontology

- If one wants to be a REALIST w.r.t. a Fundamental Physical Theory, then it must be clear what the theory is about:
  - What are the **entities** that are 'out there' in the world and and what is their mathematical representation?
- If we do not specify the ontology, the theory is only **empty mathematics**

The notion of Primitive Ontology

- By the the primitive ontology of the theory I mean what the theory is fundamentally about
- The primitive ontology is the stuff physical things are made of:
  - The wave function in GRWf and GRWm belongs to the ontology but not to the primitive ontology: according to these theories, physical objects are not made of wave functions

Primitive ontology and local beables

- Closely related to the notion of "local beables" introduced by
  - Beables: be-ables
  - "the mathematical counterparts in the theory to real events at definite places and times in the real world (as distinct from the many *purely mathematical constructions* that occur in the working out of physical theories, as distinct from things which may be *real but not localized*, and as distinct from the *'observables'* of other formulations of quantum mechanics, for which we have no use here.)" [Bell, 1976]

The common structure of BM and GRW – the PO

They both have a Primitive Ontology (PO)

- Bohmian Mechanics:
  - PO= Positions of particles
- GRW theory:
  - PO=
    - GRWf: flashes (random events in space-time)
    - GRWm: 3-d density of mass field
  - Different choices of PO define **different physical theories**

The dynamics of the PO

- It is not sufficient to specify only what is the PO: we also need to specify how it "behaves":

  What is the law of motion for the PO?
- The variables describing the **PO** must be distinguished from the other **"auxiliary" (or nomological)** variables that allow for the implementation of a dynamical law for the primitive variables

The PO and its dynamics

(what there is) & (how it behaves)

(Primitive) & (nomological) variables

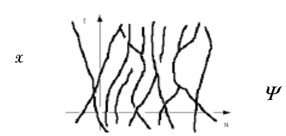
The common structure of BM and GRW – PO's dynamics

Dynamics for the PO: the wave function

- Bohmian Mechanics:
  - Deterministic evolution for Ψ (Schrödinger's equation)
  - The wave function induces a law for the PO (the guiding law)
- GRW theory:
  - The wave function evolves randomly
  - The wave function induces a law for the PO (either the mass density or the flashes)

The common structure of BM and GRW - decorations

- Dual structure: ($\mathcal{X}$; Ψ)
- $\mathcal{X}$ (=PO): "decoration" of space-time
- Ψ: governing the motion of $\mathcal{X}$



The common structure of BM and GRW – PO and its dynamics

$\mathcal{X}$ { Any Quantum Theory without Observers should be based upon a clear ontology, the **primitive ontology** PO. This primitive ontology is what the theory is fundamentally about and this is what things are made of

Ψ { There should be a quantum state, a **wave function,** whose role is to govern the behavior of the variables describing the primitive ontology. It should be regarded as a law.

PO vs Nomological Variables

- PO=output
- Nomological variables: algorithm to generate the output
  - Different algorithms can produce the very same output
    - EX: different sorting algorithms
      - Selection sort: find the minimum value in the list, swap it with the value in the first position, repeat the steps for rest of the list
      - Bubble sort: stepping through the list to be sorted, comparing two items at a time and swapping them if in the wrong order

PO and Physical Equivalence

- Theories with the same "output" are physically equivalent
- **Two theories are physically equivalent if they lead to the same histories for the PO (regardless to the evolution for the nomological variable)**

PO and Physical Equivalence

- Gauge transformation:

$$\psi \mapsto e^{i\sum_k e_k f(q_k)}\psi, \quad A \mapsto A + \nabla f$$

- Heisenberg picture:

$$\frac{dQ_i}{dt} = -\frac{1}{\hbar}\mathrm{Im}\frac{\langle\psi|P(dq,t)[H,\widehat{Q}_i(t)]|\psi\rangle}{\langle\psi|P(dq,t)|\psi\rangle}(q = Q(t))$$

- The history of the PO does not change

PO and Physical Equivalence

- Two theories are physically equivalent when they lead to the same history of the PO

or

- The PO is described by those variables that remain invariant under all possible physical equivalences

The flexible wave function

- Since what is important is the history of the PO and not the variable used to implement the law for the PO, we have a lot of flexibility:
  - Formulation of GRWf in which the wave function does not collapse
    - Physically equivalent to GRWf with stochastically evolving wave function
  - Formulation of BM in terms of a collapsed wave function
    - Physically equivalent to BM with linearly evolving wave function

Linear GRWf (GRWf without collapse)

$$r(x,i|\psi_t) = \lambda\|\Lambda_i(x)^{1/2}\psi_t\|^2.$$

$$\psi_t = \frac{\Lambda_{I_n}(X_n,T_n;t)^{1/2}\cdots\Lambda_{I_1}(X_1,T_1;t)^{1/2}\psi_t^L}{\|\Lambda_{I_n}(X_n,T_n;t)^{1/2}\cdots\Lambda_{I_1}(X_1,T_1;t)^{1/2}\psi_t^L\|}$$

$$\Lambda_{I_k}(X_k,T_k;t)^{1/2} = U_{t-T_k}\Lambda_{I_k}(X_k)^{1/2}U_{T_k-t} = U_{t-T_k}\Lambda_{I_k}(X_k)^{1/2}U_{t-T_k}^{-1}$$

$$\psi_t^L = U_{t-t_0}\psi_{t_0}$$

GRWf without collapse

$$r(x,i|\psi_t) = \lambda\frac{\|\Lambda_i(x)^{1/2}\Lambda_{I_n}(X_n,T_n;t)^{1/2}\cdots\Lambda_{I_1}(X_1,T_1;t)^{1/2}\psi_t^L\|^2}{\|\Lambda_{I_n}(X_n,T_n;t)^{1/2}\cdots\Lambda_{I_1}(X_1,T_1;t)^{1/2}\psi_t^L\|^2}.$$

- The RHS defines the conditional rate for the next flash after time t, given the flashes in the past of t
- This conditional rate defines the same flash process for GRWf
- Ψ_t^L governs the evolution of th s-t point process of GRWf. Thus, it can be regarded as a no-collapse theory involving flashes.

GRWf without collapse

$$\mathbb{P}_{\psi_t^L}(\text{future flashes}|\text{past flashes}) = \mathbb{P}(\text{future flashes}|\psi_t).$$

- The prob. distr. of the future flashes, given the collapsing Ψ_t, does not depend on the past flashes
- The prob. distr. of the future flashes, given the non collapsing Ψ_t^L does depend on the past flashes
  - The two theory should be considered as two formulations of the same theory (they are physically equivalent)

BM with collapse

$$\frac{dQ_i}{dt} = \frac{\hbar}{m_i}\mathrm{Im}\frac{\psi^{C*}\nabla_i\psi^C}{\psi^{C*}\psi^C}(Q_1,\ldots,Q_N)$$

$$i\hbar\frac{\partial\psi^C}{\partial t} = -\sum_{i=1}^{N}\frac{\hbar^2}{2m_i}(\nabla_i - i\tilde{A}_i)^2\psi^C + (V+\tilde{V})\psi^C$$

$$\tilde{A}_i = \frac{i}{\sigma^2}(q_i - Q_i), \quad \tilde{V} = -\frac{i}{\sigma^2}\sum_{i=1}^{N}\frac{\hbar^2}{m_i}(q_i - Q_i)\cdot\mathrm{Im}\frac{\psi^{C*}\nabla_i\psi^C}{\psi^{C*}\psi^C}$$

Empirical disagreement between BM and GRW !!!!

- The empirical predictions of BM are exactly and always those of QM
- The empirical predictions of GRW are not (only approximately and in most cases)
- One could empirically distinguish between BM and GRW (but no decisive test could yet be performed)

Empirical disagreement between BM and GRW ...

- This empirical disagreement is usually explained by appealing to the fact that on one theory the wave function obeys the Schrödinger evolution while in the other it does not...

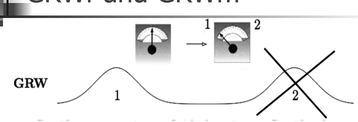
Empirical disagreement between BM and GRW ????

- But if GRW can be reformulated according to a Schrödinger evolving wave function and BM can be reformulated according to a collapsed wave function, where is the empirical disagreement between the two theories coming from?

The notion of empirical equivalence

- the empirical equivalence of two theories basically amounts to the assertion that the two worlds, governed by the two theories, share the same macroscopic appearance
- the macroscopic appearance is a function of the PO, not directly a function of the wave function:
  - the position Z of a pointer at t is a function of the PO Z=Z(PO)

Empirical agreement between GRWf and GRWm



- Consider any experiment, which is finished at time t.
- At time t, the result gets written down, encoded in the shape of the ink; more abstractly, the result gets encoded in the position of some macroscopic amount of matter
- If in GRWf this matter is in position 1, then the flashes must be located in position 1; thus, the collapses are centered at position 1; thus, the wave function is near zero at position 2; thus the density of matter is low at position 2 and high at position 1; thus, in GRWm the matter is also in position 1, displaying the same result as in the GRWf world.

The notion of empirical equivalence with QM

- the probability of the (macro) event Z_t=z agrees with the distribution predicted by QM (obtained by integrating $|\Psi_t|^2$ over all configurations in which the pointer points to z)
- If we have **macroscopic** $|\Psi_t|^2$ equivariance w.r.t. to the Schrödinger evolution then the theory is empirical equivalent to QM

Empirical equivalence and equivariance

- Macroscopic $|\Psi_t|^2$ equivariance w.r.t. to the Schrödinger evolution:
  - BM has this property:
    - it follows from the microscopic $|\Psi_t|^2$ equivariance
  - GRW does not have this property:
    - The lack of it follows from the "global warming"
- This is the source of the empirical disagreement between the two theories

The role of the wave function

- Humean view of laws:
  - The simplest and most informative way of expressing our best theory of the world
- Non-Humean view of laws:
  - "The wave function is a law"
    - Part of the laws are also masses, charges and all the other parameters appearing in the theory

Problems with the view that the wave function is a law

- Ψ evolves in time
  - Quantum cosmology suggests the universal wave function is static
- Ψ is controllable
  - Not the universal wave function
- There are different degrees of reality
  - If one is nominalists wrt laws, the wave function does not exist
  - If one is realist, it exists as an abstract entity

PO and ontological commitment

- PO: what you **need** to postulate as existing in the world if the theory is true
- Nomological variables: you can be metaphysically neutral w.r.t. to them, you **need not** be committed to their existence in order to formulate the theory (flexibility of the dynamical variable)

State, PO and supervenience

	PO	state
BM	x	(x,Ψ)
GRWm	$m(x)$	Ψ
GRWf	$F=\{(x_i,t_i)\}$	Ψ

- In GRWf, GRWm the PO is determined by the state Ψ:

  Ex: $m(x)=f(\Psi)$, Flashes = $f'(\Psi)$

Supervenience

- Supervenience of Y on X: Y=f(X)
- No differences in Y that are not differences in X
  - In GRW the PO *(m, F)* supervenes on the wave function
  - So we do not really need to add it ...?

Logical supervenience

- Y logically supervenes on X:
  - there is nothing in Y that cannot be found in X, Y is exhausted by X
    - Ex 1: water=$H_2$0;
    - Ex 2: heat=molecular motion;
    - Ex 3: mind=brain (in identity theories)

Natural supervenience

- Y naturally (or nomically) supervenes on X
  - there cannot be Y without X but in Y there is more than what is in X: there is a (physical) LAW that connects Y and X
    - Ex 1: PV=nRT
    - Ex 2: The PO in GRW naturally (not logically) supervenes on Ψ

State, PO and Supervenience

- The mass density and the flashes in GRWm and GRWf are not "hidden variables" since, unlike the positions in BM, they are determined by the wave function
- Nonetheless, they are additional elements of the GRW theory that need to be posited in order to have a complete description of the world in the framework of that theory

The measurement problem revisited

- The moral of the measurement problem is NOT the one of Bell
- Rather, it is that the wave function cannot represent physical objects
- "Bohm"-like theories:
  - PO independent on Ψ
- "GRW"-like theories:
  - PO is a function of Ψ

PO and symmetries

- Symmetries are "properties" of the law which governs the dynamics of the PO

PO and symmetries

- $P_\psi(\mathcal{X})$: (probability) law for $\mathcal{X}$
- $\mathcal{X} \rightarrow \mathcal{X}_g$ natural geometrical action of g on $\mathcal{X}$
- The law is symmetric under g if

$$P_{\psi g}(\mathcal{X}_g)=P_\psi(\mathcal{X})$$

for suitable action $\psi \rightarrow \psi_g$ of g on ψ

PO and symmetries

- Easy part: $\mathcal{X}$ transform the tight way
- Novelty: ψ is allowed to transform in any fancy way
- EX: Galilean boosts in BM

$$\tilde{Q}_i(t) = Q_i(t) + vt$$

$$\tilde{\psi}_t(q_1,\ldots,q_N) = \exp\Big(\frac{i}{\hbar}\sum_{i=1}^{N} m_i(q_i\cdot v - \tfrac{1}{2}v^2 t)\Big)\,\psi_t(q_1 - vt,\ldots,q_N - vt)$$

after $V(q_1,\ldots,q_N)$ is replaced by $V(q_1 - vt,\ldots,q_N - vt)$

PO and relativity

- The flashy ontology was invented by Bell [1987] as a step toward a relativistic GRW theory:

"I am particularly struck by the fact that the model is as Lorentz invariant as it could be in the non relativistic version. It takes away the ground of my fear that any exact formulation of quantum mechanics must conflict with fundamental Lorentz invariance."

PO and relativity

- Since symmetries concern the histories of the PO (and not the wave function) **Different** PO may lead to **different** symmetries
  - Example:
    - GRWf can be made relativistically invariant (Tumulka)
    - GRWm is NOT relativistically invariant

What is the **structure** of Quantum theories without Observers?

- There is a clear primitive ontology, and it describes matter in space and time
- There is a state vector Ψ that evolves either according to Schrödinger's equation or for a long time approximately so
- The state vector Ψ governs the behavior of the PO by means of (possibly nondeterministic) laws
- The theory provides a notion of a typical history of the PO, for ex, by a probability distribution on the space of all possible histories, from this notion the probabilistic prediction emerge
- The predicted probability distribution of the macro configuration at time t determined by the PO (usually) agree (at least approximately) with that of the quantum formalism

"That's all, Folks!"