Introduction to: “Collected Papers in Wave Mechanics” by Erwin Schrödinger

Valia Allori
Philosophy Department
Northern Illinois University

This is a collection of six papers that Schrödinger published at the rate of almost one a month in 1926. Three more papers written in 1927 were added to the second German edition of the book published in 1928, before being translated into English.

This book contains the foundation of wave mechanics as a theory of matter, in which the now-famous Schrödinger equation first appears. As it is acknowledged by the author himself in his introduction, Schrödinger wrote the first paper without knowing exactly what deeper implications it may have. It was like exploring a dark, unknown room with a flashlight: you never know whether there is actually a door until you find one. Schrödinger’s inquiry was driven by the desire of understanding what lies behind the phenomena. He believed that scientific theories should be taken as describing the mechanisms which give rise to the experimental results rather than merely systematizing them. In other words, he was a scientific realist searching for intuitive models which he believed would shed some light on the nature of things, just as a flashlight would indicate the door in the room. In his first paper in this collection, “Quantisation as a Problem of Proper Values (Part I),” his aim is to provide a deeper explanations of the ‘quantum rules’ which were merely postulated to reproduce the data. This explanation is tentative, but it suggests that there may be some oscillatory phenomenon in the atom, so that one can try to build a ‘wave mechanics’ to account for the experimental data. The other papers continue in this enterprise, as they provide the building blocks of what Schrödinger regarded as a promising model to describe reality, refining the ideas, and filling in the holes. Ultimately, however, the project turned out to be unsuccessful. In fact while in the papers in this collection Schrödinger points out at the difficulties of his project and he is hopeful of solving them, the situation quickly changed after 1928. For a variety of reasons the Copenhagen School and its anti-realist attitude had won over most physicists, and Schrödinger stopped working on quantum theory if not to criticize it. So, while one may naturally think of the first paper in this collection as a starting point, namely the birth of the new quantum theory based on

---

1 See e.g. M. Beller (1999): Quantum Dialogue: The Making of a Revolution, University of Chicago Press, for more on this.
waves, I think this can also be seen as a point of arrival, as Schrödinger did little more work on quantum theory after these papers. In any case, Schrödinger’s intuitive model of the world is a mechanics of waves, and it is interesting to see where it is coming from. By a quick look at Schrödinger’s work before this set of papers, one notices several examples of his desire to understand things intuitively. First of all, as Schrödinger himself remarked multiple times, he was strongly influenced by Boltzmann (one of his teachers at the University of Vienna was Boltzmann’s student) and his realist attitude of understanding the macroscopic phenomena in terms of the microscopic classical dynamics. For instance, just like Boltzmann, at some point Schrödinger believed that the atoms, the fundamental particles postulated in classical mechanics, actually exist, and in 1914 he even wrote a paper to support the atomic hypothesis using an analysis in terms of elastic phenomena. Also, to support that he was a realist, let me notice that in 1925 Schrödinger wrote a short philosophical essay arguing against Mach’s view that economy and efficiency are the only factors to consider in scientific investigations. So Schrödinger was a realist, but why about waves? Part of the reason can be tracked to the work of Bohr, Kramers and Salter who in 1924 proposed a theory of radiation emission in terms of ‘virtual fields.’ Schrödinger was enthusiastic about the paper because it provided a way of physically visualize what was happening in radiation emission phenomena, but he did not understand why the fields needed to be virtual. Consequently he reformulated the theory in terms of a real field instead along the lines

---

5 This essay, ‘Seek for the Road,’ has been published later in E. Schrödinger (1964): My View of the World, Cambridge: Cambridge University Press: 3-60.
of de Broglie, who complemented Einstein’s idea that to every wave there is a particle with the idea that to every particle there is a wave.\textsuperscript{8}

Another piece of the puzzle to understand how he arrived at wave mechanics comes from his reflection about Boltzmann’s statistical mechanics. In 1925 he wrote three papers of his on quantum gases.\textsuperscript{9} He wanted to give an explanation of Planck’s radiation law in terms of a gas of light quanta and by applying the Bose-Einstein statistic developed the year before.\textsuperscript{10} Schrödinger was unsatisfied with the fact that the particles in a Bose-Einstein gas are counted in a way which he regarded as physically artificial, and he thought that something in their nature must explain why. However instead of providing a microscopic description of the gas, Schrödinger proposed to treat the gas as a whole, as a matter of convenience. In particular in the third gas paper he used de Broglie’s theory to describe the gas not as a collection of particles but as matter wave. By doing this he showed that the Bose-Einstein counting procedure can be understood as a counting method for standing wave modes. Schrödinger saw this as a strong indication that there had to be something right about the matter wave hypothesis. In other words, while originally the idea of describing the gas as a matter wave was simply a useful tool to overcome the lack of understating of the nature of the gas particles, now it is seen as providing some insight on how to get such understanding: “particles are nothing more than a kind of ‘wave crest’ on a background of waves.”\textsuperscript{11} Schrödinger therefore already in this paper entertained the possibility that particles could be reduced to localized wave packets, even if he immediately realized that such a packet would quickly spread out. Be that as it may, after these papers Schrödinger wanted to see whether he could understand other experimental findings in terms of oscillatory phenomena, and started with the spectrum of the hydrogen atom. He wanted to identify the orbits as suitable modes of vibrations of a wave equation. In other words, he wanted to reproduce the energy levels of the hydrogen atom as the proper values (\textit{eigenvalues}, in modern terminology) of a suitable wave equation (hence the title of the first paper of this collection). Interestingly, since he wanted to have the

\begin{footnotes}
\end{footnotes}
new theory compatible with relativity, in December 1925 he found a relativistic wave equation, now known as the Klein-Gordon equation. However, he soon discarded because it gave the wrong results. This was the first attempt at wave mechanics, before publishing the papers in this volume.

Before leaving you to enjoy the papers for yourselves, let me point out some of their salient ingredients, telling some more history while doing that. In January 1926 Schrödinger wrote the first paper in this volume namely “Quantisation as a Proper Value Problem (Part I),” in which as we just mentioned he lays the foundations of wave mechanics. Given that the relativistic equation was not empirically adequate, Schrödinger develops a non-relativistic version which instead could account for the observed values of the hydrogen spectrum. The aim of this paper is straightforward: show that the hydrogen spectrum can be reproduced in terms of nodes of a suitably vibrating string, along the lines of de Broglie’s hypothesis of matter waves. However, the presentation is extremely abstract: there is little mention of his metaphysical hypothesis that matter is a wave. Rather, he considers a classical Hamilton-Jacobi equation for a generic particle and then instead of finding a solution for this equation, he searches for a solution of the associate variational problem, which turns out to be the stationary version of his now-famous equation. While Schrödinger writes that the successful reproduction of the hydrogen spectrum using a wave equation suggests that there is some vibration process in the atom, he however does not push for such an interpretation at this stage. Presumably because he had already in mind some potential problems for this interpretation: in addition to the already mentioned difficulty in interpreting particles as wave packet given that it quickly spreads, the wavefunction (namely the solution of his wave equation) for a system of $n$ classical particles would be a function of $3n$ spatial coordinates and therefore describe a wave in $3n$-dimensional space that could not be identified with ordinary physical space.

In his second paper, “Quantisation as a Proper Value Problem (Part II)”, published about a month after the first, Schrödinger provides a more intuitive presentation in terms of the formal analogy (discovered by Hamilton) between geometric optics and classical particle mechanics: both optics and mechanics obey the same variational principle for the same type of characteristic function. This characteristic function had to be minimized, and Fermat’s principle of the shortest time and the mechanical principle of least action are just particular cases of Hamilton’s more general principle.

---

12 He mentions the fact that the relativistic equation would lead to half-integral azimuthal quantum numbers in the first paper. A proposed relativistic equation for a single electron can be found in “Quantisation as a Proper Value Problem (Part IV)” paragraph 6. See later for more on this.

13 See L. Wessells (1979) *op. cit.*
Schrödinger uses this analogy to suggest that one needs wave mechanics as a theory of matter: when geometric optics fail, we need a wave theory in which rays need not be assumed; similarly we need wave theory when classical mechanics fails and the notion of path loses its meaning. Then he proceeds to choose his equation as the simplest equation a wave would obey. This is the same that he obtained in the first paper, but now it is coming from very different considerations, as a product of a new hypothesis about the nature of matter. Here he mentions again the possibility of interpreting the particle as a wave packet ‘so long as we can neglect any spreading,’ however granting that he has no proof that this approximation is generally valid (he will work more on this in the third paper of this collection, as we will see below). He reinforces his realist motivation by emphasizing that in this new picture one can imagine what is going on inside an atom, and therefore one can understand it. Having arrived to his wave equation through the Hamiltonian analogy, in this second paper Schrödinger applies it to other cases, such as the Planck oscillator and different varieties of rotators, with success. However he anticipates that a more complete analysis would need the development of a perturbation theory analog to the one used in classical mechanics, which he will indeed develop in the third part of his “Quantisation as a Problem of Proper Values” series, the fifth paper of the present collection. These results opened the door to the explanation of the atomic spectra of diatomic molecules.\textsuperscript{14}

The third paper, “The Continuous Transition from Micro- to Macro-Mechanics,” is where Schrödinger really works to make his mathematical model into a physical one. He explores the possibility of interpreting what we have called up to now ‘particles’ as localized wave packets. Schrödinger shows that in one dimension a group of suitable waves, intended here are the true microscopic description, can superimpose to form a well-defined and localized ‘burst’ that does not spread out, so that we could identify it with a ‘macroscopic’ particle with a definite trajectory. However, Schrödinger admits that the calculations have not been done yet for more realistic cases, for which the situation might change.\textsuperscript{15}

The issue taken up in the fourth paper, “On the Relation between the Quantum Mechanics of Heisenberg, Born and Jordan, and that of Schrödinger,” is different, but

\textsuperscript{14} E. Fues (12926) “Zur Intensität der Bandenlinien und des Affinitätsspektrums zweiatomiger Moleküle,” Annalen der Physik 81: 281-313. Fue was Schrödinger’s assistant in Zurich.

provides another piece of the puzzle that Schrödinger wanted to solve. Before Schrödinger’s wave mechanics, various attempts have been proposed to account for experimental data such as the atomic spectra. In 1925, three papers emerged from the collaboration of Heisenberg, Born and Jordan\(^{16}\) (and the idea was immediately taken up by Dirac\(^{17}\)) that allowed to describe the experimental results in a general way in terms of infinite matrices, the so-called matrix mechanics. Both matrix mechanics and wave mechanics can account for the experimental outcomes, but they are very different from one another on many levels. In his paper Schrödinger points out some differences (continuum vs. discrete picture, algebraic vs. differential equations), but quite strangely he is not explicit here in pointing out what in other correspondence he identified as the difference that mattered to him the most, namely that his wave mechanics is the only one which is amenable to a realist interpretation. In contrast with matrix mechanics, his view has the virtue of being visualizable (anschaulichkeit): one can see, or imagine, what happens into the atom. The aim of Schrödinger in this paper is to show that these two theories are equivalent reformulation of the same reality. He does that by showing that for each operator in wave mechanics there is one corresponding infinite matrix.\(^{18}\) Proving this equivalence is important to him because, all other things being equal, the visualizability of his wave mechanics could be taken as a reason to prefer it over matrix mechanics. Indeed, this is the preference expressed by Lorentz after he read the first two of Schrödinger’s papers,\(^{19}\) and this appreciation led to Schrödinger’s invitation to the 1927 Solvay conference he was organizing.\(^{20}\) In addition of expressing excitement, Lorentz however also expressed doubts about wave mechanics, in particular he objected that a wave in configuration space could be thought as a physical wave.


\(^{19}\) K. Przibram (1967) op. cit.

Schrödinger was well aware of that, as it is shown by the fact that he acknowledges this problem again at the end of this paper.

Schrödinger nonetheless is hopeful of solving these problems, and continues in his quest for an intuitive understanding of the quantum world in the fifth paper of this collection, “Quantisation as a Problem of Proper Values (Part III).” In this article Schrödinger develops the theory of perturbation mentioned in Part III to apply the theory beyond directly soluble problems. He discusses the Stark effect, the shifting and splitting of Balmer spectral lines (the most intense lines of the Hydrogen spectrum) due to the presence of an external electric field. He does not discusses the Zeeman effect, an analogous splitting due to a magnetic field, because as discussed in the first paper magnetic effects, which he connects to relativistic effects, cannot yet be accounted for.\(^\text{21}\)

The next paper, “Quantisation ad a Problem of Proper Values (Part IV),” marks another stepping stone, as it contains the Schrödinger wave equation in its most general form, a proposal for a relativistic wave equation, and a new proposal on how to think of the wavefunction which would solve the problem of being realist about a wave in configuration space. The stated aim of the paper is to generalize the equation proposed in the previous papers, which holds only for a fixed value of energy and it is time-dependent only through the phase. As such, Schrödinger claims, it is not really a vibration equation, so he embarks in eliminating the energy from his previous equation. By doing that he first finds a fourth order equation. Aside from the complexity of it, he observes that, given that for time independent potentials the equation is equivalent to the product of two second-order equations, one could instead consider a fundamental wave equation of the second order instead. This is the Schrödinger equation which was about to become famous. Nonetheless, he notices, the price to pay for this is that the solution of this equation has to be complex. After generalizing perturbation theory to this case, in paragraph 6 Schrödinger constructs a relativistic equation for the single electron by substituting quantities with operators. He remarks that more justification needs to be provided for this equation, not only because needs to be corrected for spin, but also because he did not want to rely on such a formal analogy, justified only in virtue of the fact that it would reproduce the nonrelativistic equation with the right Hamiltonian. He therefore moves to the physical significance of the solution of his equation. To respond to the objection that a field in configuration space cannot be considered physically vibrating and convinced that the wavefunction had some

---

\(^\text{21}\) This effect will be later accounted for by Fock, who develops a relativistic wave equation: V. Fock (1926): “Zur Schrödingerschen Wellenmechanik,” Zeitschrift für Physik 28: 242-250.
electromagnetic meaning, he proposes that the square of the wave function is to be interpreted as a charge density, reinforcing the idea that what we call ‘particles’ are actually localized wave packets.

The last three papers were written in 1927 and constitute further development of Schrödinger’s project. The first of these papers, On the Compton Effect,” provides a partial answer to a criticism that he had received by many, including Bohr and Heisenberg during his visit to Copenhagen in October 1926, namely that wave mechanics, being a theory of continuum, cannot account for the distinctive feature of quantum theory, namely that phenomena are discrete. The idea was that the photoelectric effect (when a material emits charged particles after absorbing radiation) and the Compton effect (the frequency shift of electromagnetic radiation after interaction with matter) could only be explained in terms of light quanta.\(^\text{22}\) In response, Schrödinger provides an account of the phenomenon in terms of wave mechanics. He follows the work of Gordon, which developed a relativistic wave equation and already described the Compton effect in wave-mechanical terms,\(^\text{23}\) but in a less technical way. Schrödinger argues that the incoming radiation is diffracted on a standing ‘charge density’ wave, namely the electron, just as light is diffracted on a standing acoustic wave. Around the same time Wentzel\(^\text{24}\) proposed an analysis of the photoelectric effect within wave mechanics, and later Mott developed a general method to account for any kind of discontinuous phenomena using wave mechanics, clearing out the road to Schrödinger regarding the objection that one needs a discrete ontology to account for quantum phenomena.\(^\text{25}\)

Also the last two papers in the collection use the relativistic extension of the wave theory. In the second to last, “The Energy-Momentum Theorem for Material Waves,”


Schrödinger wants to see whether one can blend together quantum theory and classical electrodynamics using the Hamilton principle that was used by some to derive the relativistic wave equation, and emphasizes many of the difficulties on encounter in doing such a project (the theory gives the wrong results when applied to the hydrogen atom). In the final paper of this volume, “The Exchange of Energy according to Wave Mechanics,” Schrödinger wants again to defend his view against the criticism that his theory cannot account for ‘quantum jumps.’ Schrödinger thus shows that two atoms in resonance exchange energy as if they were exchanging discrete quanta. He proposes that we should re-understand quanta in terms of wave frequency, and that the notion of resonance is key to understand quantum phenomena. Also, he provides a wave-mechanical demonstration of Planck’s radiation law without postulating the existence of light quanta. He acknowledges that one could interpret these results in terms of probablity field rather than material fields, as Born had just done, but he rejects it somewhat cryptically on grounds that it is not sufficinelly warranted. It does not seem that he disliked determinism: early in his career he supported such a view, as shown by his work on the BKS theory of radiation, where momentum and energy were conserved only statistically. Rather, the problem with Born’s statistical interpretation of the wavefunction seems to be that one does not gain much from it, as a matter of explanation. He does not elaborate on what the problem is supposed to be in this paper but in a letter to Planck he writes: "What seems most questionable to me in Born’s probability interpretation is that […] the probabilities of events that a naïve interpretation would consider to be independent do not simply multiply when combined, but instead the 'probability amplitudes interfere' in a completely mysterious way (namely, just like my wave amplitudes, of course).” In this way Schrödinger anticipates the strongest objection to the epistemic views of the wavefunction: how can one explain the real phenomena of diffraction and interference if the wavefunction represents our state of knowledge of reality, rather than reality itself?

This paper concludes the collection, both practically and pragmatically, as it marks the end of Schrödinger realist project. In 1927 Heisenberg wrote his celebrated uncertainty principle paper, in which his main goal was to defend his matrix mechanics from Schrödinger’s charge of lack of visualizability. He therefore endorses a partcle picture of reality, and argues that by abandoing the idea of trajectory then everything becomes

26 O. Klein, op. cit.; T. de Donder, H. van der Dungen op. cit.
28 N. Bohr, H.A. Kramers, J.C. Slater (1924), op. cit.
comprehensible and intuitive. In this way he de facto nullifies Schrödinger’s proof of equivalency between matrix and wave mechanics, as they are both visualizable. Incidentally, Heisenberg’s paper made Bohr very upset, because he wanted to keep the wave picture together with the particle picture.³⁰ In fact Bohr disliked the light quanta and used wave concepts in his theories: he developed the BKS theory to dispense of light quanta, and his atomic theory in terms of stationary states was more compatible with Schrödinger’s wave theory than the particle theory favored by Heisenberg. However, Bohr insisted in reading Schrödinger’s picture instrumentally, and began to develop what was to become his theory of complementarity: wave and particles are concepts which complement each other, and show their face one at a time. As a result, he strongly pressed Heisenberg to change his mind and endorse his view, so that the ‘Copenhagen-Göttingen school’ formed a united front against Schroedinger’s wave picture. The establishment of the Copenhagen dogma culminated in 1928, when Bohr delivered the now famous Como lecture, which provides the basis of the complementarity view. On another front Dirac³¹ simply side-stepped the question of the relative physical realism by introducing wave and matrix mechanics as calculational aids and emphasizing his own symbolic approach for its ability to express the physical laws in a neat and concise way. Eventually Schrödinger changed research topics to study a way of unifying quantum theory and relativity using fields, along the line of research developed by Einstein, with which he created a long and fruitful collaboration. He returned to quantum theory sporadically in print, with the notable exception of 1935, when after the publication of the Einstein-Podolsky-Rosen paper³² he wrote a paper entitled “The Present Situation in Quantum Mechanics”³³ in which, among other things, he presented his now famous cat which is impossibly dead and alive as a reductio ad absurdum for the theory.

³¹ P. A. M. Dirac (1925), op. cit.