Primitive Beables are not Local Ontology: On the Relation between Primitive Ontology and Local Beables

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Abstract

When discussing quantum ontology, the debate has recently focused on comparing and contrasting wavefunction realism and its rivals. Among them one finds the primitive ontology approach, which is often conflated with the local beables program. In this paper I wish to clarify what I take to be the distinction between the notion of primitive ontology and the one of local beable. I argue that the primitive ontology is the local beable which allows for a dynamical, constructive explanation which preserves symmetries.

1. Introduction

Quantum mechanics has been at the center of the philosophical debate between realism and antirealism for a long time. According to the scientific realist, theories gives us our best shot at understating the nature of reality, while the antirealist believes that they are mere instruments to conveniently systematize the data and predict experimental results. In 1926, the 'fabulous year' of quantum theory, first Schrödinger and then Heisenberg found two incompatible ways of accounting for the current experimental data. While Heisenberg's matrix mechanics seemed impossible to reconcile with a realist view, Schrödinger's wave mechanics was developed exactly to make a realist reading possible. Anyway, mostly for sociological reasons,[2] after the 1928 Bohr Como lecture, instrumentalism became the orthodoxy, and quantum mechanics was considered impossible to reconcile with scientific realism, even given Schrödinger's and Einstein's efforts to the contrary. These culminated in 1935, with Schrödinger's cat paper in which he presents the measurement problem, and Einstein's paper which he wrote with Podolsky and Rosen to (try to) show the incompleteness of quantum theory. After 20 years or so of stasis, in 1950s the situation started slowly to change, as new proposals to solve the measurement problem came to life. Thus, people started taking seriously (at least) three of the solutions of the measurement problem: the pilot-wave theory, the spontaneous localization theory, and the many-worlds theory. They provide different pictures of the world: in the pilot-wave theory the world is made of particles and fields, while in the many-worlds and the spontaneous localization theories matter is made of a field, which evolves differently in the two theories.[3] Until fairly recently, however, it was not clear that, even focusing on one

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[2] See Beller (1999), Cushing (1994).

[3] Initially these theories were called 'interpretations' of the quantum formalism, because realists took quantum theory at face value and thus tried to understand what the mathematics of the theory corresponded to in the world. However, the fact that (with the exception of many-worlds) the other theories have a different formalism finally

of these theories, the same formalism may be interpreted as to give us different ontologies. This is where the debate has shifted in the last decade.[4] That is, recently the discussion has focused on how to 'interpret' these 'interpretations.' Some people endorse *wavefunction realism*, the view that the wavefunction, the main mathematical object in each of these theories, represents material objects.[5] In contrast, proponents of *local beables*[6] argue that each theory needs to have an ontology in three-dimensional space. Then, there is the *primitive ontology* program,[7] which many have taken as substantially identical to the local beable approach.[8] In this paper I wish *to distinguish the two notions*: I argue that even if they share many similarities, requiring a theory to have a primitive ontology is *stricter* than requiring it to have local beables: some local beables do not make good primitive ontologies. Both approaches come from a realist attitude: a theory is more than just an instrument to predict experimental result. First, it needs to be clear what the theory describes the behavior of, namely matter, and what the laws are. In the words of Bell, there must be entities, matter and its laws, which objectively exist: beables, rather than observables. Moreover, the beables describing matter need to be *local*: the theory needs to describe the behavior of something in three-dimensional space. When exploring the reasons for this last requirement, one sees the first difference between the two approaches: while supporters of local beables talk about the direct accessibility of the local beables (as discussed in section 2), the primitive ontologists focus instead on explanatory power and symmetries (see section 3). Indeed, the primitive ontology program requires not only that the ontology is clear, and it is in space-time like local beables, but also that the explanation provided by the theory is dynamical, compositional and respects symmetries (as presented in section 4). As I conclude in the last section, failing to distinguish between the two positions may lead to important misunderstandings, while keeping this distinction in mind allows to look at the various approaches from a more informed point of view.

2. Local Beables

As anticipated, the notion of local beables comes from Bell (1987): "these are the mathematical counterparts in the theory to real events at definite places and times in the real world (as distinct from the many purely mathematical constructions that occur in the working out of theories, as distinct from things which may be real but not localized, ...)." This notion has two components: one underlying a locality feature, and the other emphasizing an existence claim. Let's start with the latter. The locution 'beable' has been introduced in explicit contrast with 'observable,' commonly used in quantum theory. The idea is that a theory should not talk about what we can observe but instead it should talk about what exists. The question should be: "what are the 'be-ables'?" rather than: "what are the 'observ-ables'?" To promote a realist understanding of quantum theory, in contrast to the dominating instrumentalist attitude, one needs to be clear about what the ontology, the beable, of quantum theory is. The main equation of the quantum formalism, the Schrödinger equation, describes the wave-like evolution of an object called the

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convinced many people to call them, more appropriately, 'theories.' To be clear, there is no harm in calling them 'interpretations,' as long as it is understood that they are really not.

[4] See Albert and Ney (2013).

[5] Most prominently, see Albert (1996, 2015), Ney (2012, forthcoming), North (2013), Lewis (2016) and references therein.

[6] This locution has been introduced by Bell (1987).

[7] Dürr *et al.* (1992) first use this idea in the framework of the pilot-wave theory, and later Goldstein (1998) informally defends it. Allori *et al.* (2008, 2011, 2014) further elaborate it and extend it to all quantum theories. Allori (2013 a, b) applies this approach to all fundamental physical theories.

[8] See e.g. Esfeld (2014a), Myrvold (2017), Maudlin (2019).

wavefunction, ψ. However, wave can superimpose, as they have finite amplitudes at different locations, and they spread out when evolving. So, they cannot reproduce localized phenomena, like the tracks observed in detectors, usually interpreted as particles' trajectories. Rather, a wave theory predicts macroscopic superpositions for everything, including stuff we observe to be localized. This is the *measurement problem*: macroscopic superpositions need to be eliminated. One way to fix this is to talk about measurements: "observable X is in an indefinite superposition state; only after a measurement X acquires definite values." However, what is a measurement? Why is it not just another physical process? A theory defined as such is vague and imprecise, and thus unsatisfactory. According to Bell, one way of precisely eliminating the macroscopic superpositions is to postulate that there are indeed particles, which are always localized by definition, and which are guided in their motion by the wavefunction (as assumed in the pilot-wave theory). Another option is to modify the evolution of the wavefunction as to forbid macroscopic superpositions (like in the spontaneous localization theory). Or one may grant that macroscopic superpositions exist, but, since the branches corresponding to localized outcomes effectively do not interact, we experience them as localized (as in the many-worlds theory). Given that these theories no longer invoke the notion of measurement in their definition, they are precise in their ontology: except for the pilot-wave theory in which there are also particles, the beable of these theories is the wavefunction. More precisely, w*avefunction realists* argue that one can read-off the beables from their formalism: they are those elements of the theory which appear in its most fundamental equations. Being the wavefunction the beable (or part of the beables) of these theories makes wavefunction realists rethink of the meaning of 'real world' in the quote at the beginning of this section. In Newtonian mechanics one straightforwardly understands the real world (or physical space) as mathematically represented by three-dimensional Euclidean space: this is where the ontology, in this case particles, live. In quantum mechanics however, this is not the case. The wavefunction is best understood as a field in configuration space, the space of all the accessible configurations.[9] Since each configuration is three-dimensional and since it is *as if* there are N of them, where N is of the order of 10^{23}, configuration space is a high-dimensional space. If the wavefunction is the beable of the theory, then this space is the mathematical representation of 'the real world.'

The 'locality' part in 'local beables' roughly corresponds to the idea that what exists in a given region of the 'real world' does not depend on what happens in another region. Thus, the wavefunction *is* a local beable in configuration space.[10] However, if someone thinks that local beables are entities in three-dimensional space, rather than any other physical space, then clearly the wavefunction is not a local beable.[11] If so, then quantum theories like many-worlds and spontaneous localization, which have only the wavefunction, need to have a new (three-dimensional) local beable. This is (part of the reason) why theories like GRWm, GRWf, Sm, Sf and varieties of 'GRWp' and 'Sp' have been proposed.[12] The 'm'-type theories (GRWm and Sm) are to be understood respectively as the modification of the spontaneous localization theory and of the many-worlds theory so that the theory describes the evolution of a matter density field m (which happens to be defined in terms of the wavefunction). This matter field is the local

[9] These configurations are the positions that particles would have, if there were any. In quantum theories, except the pilot-wave theory, there are no particles, but the wavefunction ψ can be written *as if* it depends by their configurations $\psi(r_1, r_2, \dots, r_N)$.

[10] See Albert (1996), Ney (2015).

[11] See Goldstein *et al.* (2011), Maudlin (2019).

[12] See Bell (1987) for GRWf, and Ghirardi *et al.* (1995) for GRWm. Allori *et al.* (2008, 2011, 2014), Allori (2019), Allori (2020 a) discuss many varieties of these theories, including Sm, Sf, and Sp.

beable of these theories: it is what matter is made of. In the ‘f’-type theories (GRWf and Sf) instead the local beable is given by ‘’flashes,” space-time events, while ‘p’-theories (GRWp-type theories and Sp-type theories) have a particle ontology.

I will dub defenders of local beables in this sense *localists*, to contrast them with wavefunction realists.[13] Localists include primitive otologists, presented in the next section, and quantum state primitivists, discussed below, as well as others. Wavefunction realists and localists are both realists, so they agree on the necessity for a theory to have a precise ontology, in contrast with instrumentalist readings of quantum theory. However, they disagree on what counts as a good ontology: wavefunction realists allow it to live in any type of dimensional space, while localists restrict them to three-dimensional space. Consequently, they disagree about which type of locality is important: while localists think most of three-dimensional locality (3-locality), wavefunction realists are happy of having a local theory in configuration space ($3N$-locality).

But why do localists require local beables to be three-dimensional?[14] Maudlin argues that "to give the theory empirical content, we need some sort of items that exist and move in physical space,[15] influenced by the quantum state," and that "the structure of the wave function must be projected down from configuration space into physical space."[16] Moreover, by having local beables, the connection between the theory and the world is direct and transparent in a way that would not be possible otherwise: "Collections of atoms […], because they are local beables, can unproblematically be rock-shaped and move in reasonably precise trajectories."[17] Without local beables, instead, one needs some mapping from configuration space to the three-dimensional space we perceive: the world we experience must suitably emerge from the description in configuration space. However, localists argue, there is no preferred mapping, and there is nothing to suggest dimensions are grouped into threes, as needed if one wants a point in configuration space to correspond to particle configurations in a three-dimensional space. In addition, different mappings may give rise to incompatible metaphysics: in that case, which beable will actually emerge?[18] Moreover, there is the problem of empirical coherence. A theory is said to be empirically incoherent in case its truth undermines our empirical justification for believing the theory to be true. Arguably, any theory gets confirmation by spatiotemporal observations. The claim is that, in contrast with a theory with local beables, wavefunction realism is empirically incoherent because its fundamental entities are not spatiotemporal.[19] Furthermore, some have emphasized that wavefunction realism obscures the role of spacetime in the theory. If one formalizes the theory too abstractly, then one loses understanding: "Thinking about quantum mechanics in terms of a wavefunction on configuration

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[13] The alternative locution ‘local beablists,’ which perhaps is more accurate, sounds horrible to me. If you wish, alternatively you can call them ‘three-dimensionalists,’ as their focus is not much on locality (which changes meaning depending on which space one thinks is fundamental) but on the existence of an ontology in three-dimensional space, as opposed to one in a high-dimensional space.

[14] From now on, let me call local beables in three-dimensional space simply ‘local beables,’ as this is how the terminology is commonly used in the literature, even if to avoid confusion I should write 3-local beables to denote the one used by localists, and $3N$-local beables the ones of wavefunction realists.

[15] He means three-dimensional space here.

[16] Maudlin (2019), p. 110 and p. 116.

[17] Maudlin (2007).

[18] See Monton (2002, 2006, 2013), Maudlin (2007), Allori (2103 a, b). See Albert (2015), Ney (forthcoming) for discussions and responses.

[19] For a first formulation, see Barrett (1999). Also, see Healey (2002), Maudlin (2007). For responses, see Huggett and Wüthrich (2013) and Ney (2015).

space is rather like thinking about classical mechanics in terms of a point on phase space. In both cases, there is a far more perspicuous way to understand the theory, one which is connected to spacetime in a more direct way."[20] Finally, even if this is not closely connected with the existence of local beables, people have argued that it is difficult to extend wavefunction realism to relativity.[21]

Regardless, if one requires a theory to have local beables the question moves to the nature of the wavefunction. Notice that the distinction between epistemic and ontic views of the wavefunction is not very helpful here. [22] Ontic conceptions see the wavefunction as objectively existing, while epistemic approaches take the wavefunction as representing the state of knowledge of an observer. While epistemic views seem more suitable for instrumentalist approaches, wavefunction realists, primitive ontologists, and other localists all think of the wavefunction in ontic terms. However, they disagree about how exactly to think of it. Norsen proposes to eliminate the wavefunction altogether, due to its nonlocality, and to construct a theory *of exclusively local beables*.[23] He has proposed a reformulation of the pilot-wave theory in which the wavefunction is replaced by local fields, similarly to the case of classical electrodynamics, in which there are particles and local fields (however, see section 4). However, in this case it is necessary to add an infinite number of local fields, and this is presumably (part of the reason) why some other localists have proposed alternative approaches. Some suggested to take the wavefunction as a *multi-field,* or a 'poly-wave,' in three-dimensional space. This is a generalization of an ordinary classical field: as the latter specifies a value for each three-dimensional location, the multi-field of an N-'particle' system specifies a value for the N-tuple of three-dimensional points.[24] Alternatively, *quantum state primitivism* postulates that the wavefunction is a new kind of beable, which is nonlocal. It represents the quantum state, which is a novel feature of reality without classical analog.[25] In addition, *spacetime state realism* understands the wavefunction as characterizing, in terms of the reduced density matrix, some features of spacetime regions.[26] The peculiarity is that this local beable is non-separable: the whole is different from the sum of its parts. Other approaches which are often proposed by defenders of the primitive ontology approach are discussed in the next section.

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[20] Wallace and Timpson (2010).

[21] See Myrvold (2015), Wallace (2021) for the problem, and Ney (2019, forthcoming) for replies.

[22] This distinction has been proposed by Spekkens (2007).

[23] Norsen (2010), see also Norsen *et al.* (2015).

[24] This view has been defended by Hubert and Romano (2010), taking up an idea of Forrest (1988) and Belot (2012). See also Chen (2017) and Romano (2020).

[25] Maudlin (2013, 2019).

[26] Wallace and Timpson (2010).

3. Primitive Ontology

The literature on quantum ontology almost never clearly emphasize the difference between local beables and primitive ontology.[27] However, I am convinced that the two ideas are importantly distinct.[28] In this paper I aim to show that a primitive ontology is a special type of local beable.

First, the primitive ontology approach is a distinctive realist understanding of what physical theories are, and how they are supposed to be explanatory. According to the primitive ontologist, the realist enterprise does not proceed by interpreting the theoretical formalism *a posteriori*. Rather, a metaphysical hypothesis about the nature of matter is *a priori* postulated by choosing the simplest, as there is no reason to do otherwise. Since we perceive a three-dimensional world, the simplest material ontology is in three-dimensions. If this is explanatory, there is no reason to look further.[29] This also clarifies why primitive ontologists think of highly abstract mathematical objects as not suitable to represent physical objects.[30] This goes back to the late 1920s, when Lorentz, de Broglie, Heisenberg and Einstein expressed perplexities about considering the wavefunction as a material field.[31] Part of the problem was that doing so requires giving up everything we know from classical physics about explaining the phenomena.

So, the requirement of constructing the simplest theory possible is fundamentally tied to simplicity of explanation. Why should one depart as little as possible from the classical way of understanding? Becasue, why not? In football terms, there is no point in changing a winning team. Or, as the Irish say: "if it is not broken, why fix it?" The primitive ontology approach urges therefore to retain the compositional and dynamical explanation of the phenomena typical of the classical theories.[32] Thus, we should avoid

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[27] This is unfortunately the case also among some proponents of the primitive ontology approach. For instance, Esfeld (2014 a) writes: "The primitive ontology consists in one actual distribution of matter in space at any time (no superpositions), and the elements of the primitive ontology are localized in space-time, being 'local beables' in the sense of Bell (2004, ch. 7), that is, something that has a precise localization in space at a given time." See also Tumulka (2016): "The 'primitive ontology' of a theory (more or less what Bell called the 'local beables') are the variables in the theory that represent matter in space– time." However, see Allori (2015 a) for the suggestion that the two notions are not identical.

[28] This qualifier seems necessary as I am not the only defender of the primitive ontology approach, and obviously cannot speak for the others.

[29] See Allori (2013 a, b) for this argument in the framework of the primitive ontology approach. See also Emery (2017) for an independent but similar argument.

[30] See Norsen (2010), Allori (2013 a, b).

[31] See Bacciagaluppi and Valentini (2009) for a very interesting discussion of this and other issues during the 1927 Solvay Conference. Interestingly, this argument can be tracked down historically also to Heisenberg, who very vividly said to Bloch (1967), referring to configuration space realism: "Nonsense, […] space is blue and birds fly through it," to express the ultimate unacceptability of building a theory in which there was no fundamental three-dimensional space. Similar concerns have been expressed by Lorentz, who in a 1926 letter to Schrödinger wrote: "If I had to choose now between your wave mechanics and the matrix mechanics, I would give the preference to the former, because of its greater intuitive clarity, so long as one only has to deal with the three coordinates x, y, z. If, however, there are more degrees of freedom, then I cannot interpret the waves and vibrations physically, and I must therefore decide in favor of matrix mechanics" (Przibram 1967). Similarly, Schrödinger wrote: "The direct interpretation of this wave function of six variables in three-dimensional space meets, at any rate initially, with difficulties of an abstract nature" (1926, p. 39). Again: "I am long past the stage where I thought that one can consider the w-function as somehow a direct description of reality" (1935). This is also a concern heartfelt by Einstein, who expressed this view in many letters, e.g.: "The field in a many-dimensional coordinate space does not smell like something real" (Einstein 1926).

[32] In contrast, in wavefunction realism the traditional explanatory schema of the behavior of macroscopic entities in terms of microscopic ones needs to be heavily revised. Contrarily to the classical case, tables and chairs are not

Kuhnian classical-to-quantum revolutions, which would have us unnecessarily re-think the notion of scientific explanation.[33] If we postulate the fundamental building blocks of matter to be three-dimensional and microscopic as in the classical picture, then we can think of macroscopic objects as composed of them. These material Lego bricks are the primitive ontology of the theory. In terms of Einstein's distinction between constructive and principle theories,[34] primitive ontologists require constructive theories.[35] In fact, principle theories are formulated in terms constraints on physically possible processes: the principles. Think for instance of the principle that 'no perpetual motion machines can exist' used in thermodynamics. Instead, in constructive theories one derives dynamically macroscopic objects and their behavior in terms of their microscopic constituents: again, think of kinetic theory, which reduces the behavior of gases to the motion of atoms.[36] Primitive ontologists prefer constructive theories because regularities and constraints over the possible experimental findings lack explanatory power from their perspectives: we still do not know why these constraints obtain. They think that a satisfactory explanation is provided only by a dynamical, constructive theory, in which one is told the microscopic story, provided by the primitive ontology, giving rise to the observed behavior, both in the classical and the quantum case.[37]

This way of understating explanation fits well with thinking of the wavefunction as nomological. That is, it is not something that describe a material field, rather it an ingredient needed to construct the law of evolution of the primitive ontology.[38] The *wavefunction is a beable* in the sense that it expresses some objective facts rather than states of knowledge. Therefore, it is *ontic without being material*. Depending on what one thinks does the explanatory work, this nomological character of the wavefunction is either conceived as being a property of matter or as being (part of) a law of nature. One may think that glass breaks because it has the dispositional property of 'fragility.' In this way the wavefunction is seen as a property that accounts for matter's behavior under the various circumstances.[39] However, I find dispositions mysterious: what are they? How many should we postulate in order to account for the phenomena? Better, I believe, is to characterize the role of the wavefunction in terms of laws of nature.[40]

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macroscopic three-dimensional objects composed of microscopic three-dimensional particles. Rather, they are macroscopic three-dimensional objects 'emerging from' a high-dimensional wavefunction.

[33] See Allori (2015 b) for this argument.

[34] Einstein (1919).

[35] For an elaboration on this, see Allori (2020 b, manuscript a).

[36] Einstein introduced the principle/constructive distinction to express *his own dissatisfaction* of his 1905 relativity theory, formulated in terms of principles. He thought that one uses principle theories when constructive theories are either unavailable or too difficult to build. See Brown and Timpson (2006).

[37] This attitude makes clear why the primitive ontologists, given their commitment to explanation, care about locality in three-dimensional space, rather than more abstract senses of locality as the one proposed by wavefunction realists. In fact, a nonlocal world is certainly disruptive of the schema of explanation used in classical theories, which the primitive ontologist tries to preserve. The locality assumption was the one that Einstein thought would be undeniable, not necessarily because it conflicts with relativity theory but more importantly, I think, because it is in tension with our natural way of understanding the phenomena. Indeed, it is curious that wavefunction realists care about locality at all: on the one hand, they insist that we have to change our ordinary way of understanding the world (allowing it to be a high dimensional space) and then we are supposed to preserve locality in that space. But if the principle is: "Reject intuitions, go where the formalism takes you," *why should we care about locality*?

[38] See Allori (2013 a, b), Goldstein and Zanghì (2013).

[39] For this approach, see Monton (2013), Esfeld *et al.* (2014), Suarez (2015).

[40] The first proponents of this view are Goldstein and Zanghì (2013). See also references therein. For Humean variants, see Miller (2014), Esfeld (2014b), Bhogal and Perry (2015), Callender (2015). Notice that the quantum law of nature now is *not* the Schrödinger equation, which is an equation for the evolution of the wavefunction. Rather, it is a

To sum up, a fundamental physical theory possesses an explanatory architecture at the foundation of which one finds what in the theory represents matter, namely the primitive ontology. It is at the foundation because this is what the theory is wishing to explain the behavior of. In order to do so, the theory has layers which fit together for this explanatory purpose. Thus, in addition to the primitive ontology, many other variables appear in the theory to define, more or less directly, how matter moves. The primitive ontology must be a local beable: it must exist ('beable') in three-dimensional space ('local'). However, *being local is not enough for being material* (i.e., to represent what physical objects are made of): what determines whether something is material or not has to do with the role the variable has in the theory. In other words, it depends on the level in the explanatory architecture of the theory this variable occupies. Is this local beable at the foundation? In other words, does this local beable represent the entity the theory explains the behavior of? If so, this local beable is (part of) the primitive ontology (matter). Does it enter instead in the definition of how matter moves? Or, doe this local beable appear in the laws governing the behavior of matter? If so, then this local beable belongs to an 'explanatory' level. While it does exist (beable), it exists as part of the laws, broadly speaking, and not as a material entity. Thus, *local beables may be nomological*: they are beables (they exist objectively), they are local (they are in spacetime), but they are not (part of) the primitive ontology (they do not represent material objects). I think that a comparison with potentials may be useful: they are local beables (they are objective mathematical entities defined in three-dimensional space), but they do not represent anything material. Rather, like electromagnetic fields, they are another way of expressing the interaction between material things, and so they are best seen as part of the law than as part of the material ontology (I will further discuss the analogy with electromagnetic fields in the next section).[41] So, in the primitive ontology framework the ontology, namely what exists objectively (as opposed to what describes an epistemic attitude of someone), may be thought as follows: there is matter, described by the primitive ontology, and there are the laws governing the behavior of matter, described by the non-primitive, or nomological variables. Sometimes this is schematically exemplified as: $O = (PO;\ NO)$, where O is the ontology (what exists), PO the primitive ontology (matter), and NO, the *nomology*, namely the dynamical structure needed to account of the behavior of the PO (the law).[42] The semicolon which separates the material ontology (PO) from the nomology (NO) divides the roles of the variables in the theory: both ontology and nomology exist, but not in the same way. One is material, the other is not.[43]

The last ingredient of the primitive ontology approach is the importance of symmetry properties, which are taken as guiding tools for theory construction: take as primitive ontology the simplest microscopic ontology and use symmetries to find the simplest law allowing for a dynamical explanation of the

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law that the primitive ontology obeys and that describes how it moves. For instance, in the case of the pilot-wave theory, the quantum law is the so-called guidance law, which is an evolution equation for the particles' position. This law contains the wavefunction, which has no straightforward physical representation other than being an ingredient in the law. I have recently argued that the best characterization of the wavefunction is in terms of a functionalist account, which combines the advantages the nomological view as well as of the epistemic approaches, without suffering from their weaknesses (Allori, manuscript b).

[41] See an interesting discussion on the ontological status of electromagnetic fields and the Aharonov-Bohm effect in Maudlin (2018).

[42] A clarification: in cases like classical electrodynamics, in which one distinguishes between matter and light, the primitive ontology is to be contrasted with the nomology: the primitive ontology is whatever physically exists, like matter and light, as opposed to what nomologically exists, like the laws of nature. See section 4.2.

[43] See Allori (2015 a) for more on this aspect of the primitive ontology approach.

phenomena.[44] Thus, symmetries are thought as the invariances of the law for the primitive ontology.[45] This means that, for instance, in the pilot-wave theory (which I often use as example because it is the simplest and better known quantum theory with a primitive ontology) the symmetries of the theory are the symmetries of the guidance equation, *not* of the Schrödinger equation. So, a theory is invariant under a given symmetry S if the temporal evolution of the primitive ontology does not change under the transformation corresponding to S. Making the symmetries a business of the primitive ontology allows to explain why the wavefunction transforms the way it does under symmetry transformations in order to make the theory invariant (see section 4).

To summarize the discussion in this section, the primitive ontology has three roles:

1) A *metaphysical* role: it defines what materially exists at the microscopic level in three-dimensional space;
2) An *explanatory* role: it explains macroscopic phenomena dynamically and constructively;
3) A *physical* role: it defines symmetries as invariances of its law of evolution.

Therefore, primitive ontologists are stricter than other localists in the criteria for a desirable theory: not only they want a precise ontology (as wavefunction realists), not only they want a three-dimensional ontology (as localists), but also they want constructive, dynamical explanations and they want to keep as many symmetries as possible. More explicitly, a *primitive ontology is the simplest local beable that allows for a dynamical, constructive explanation which preserves symmetries.*[46]

4. Comparison

Localists advocate for a three-dimensional ontology: this is the metaphysical role of the primitive ontology above. However, a local beable may not be a suitable primitive ontology if it does not provide a dynamical explanatory schema (the explanatory role) or does not preserve symmetries (the physical role). Let's compare the two views more in detail, focusing first on the reasons why an ontology in three-dimensional space is considered desirable.

4.1. Familiarity of Explanation

The explanatory role is partly why I, as primitive ontologist, find quantum state primitivism unsatisfactory. In fact, postulating the quantum state as a new category to explain the phenomena (when other options are available) is at odds with the desire of explanation motivating the scientific realist, including the quantum state primitivist: how do we truly explain the phenomena if we invoke some mysterious objects whose nature we postulate to be unanalyzable? Maudlin claims that those who have tried to figure out what the quantum state is have succumbed to the "misguided desire to liken the quantum state to anything we are already familiar with."[47] While it is true that everybody needs starting

[44] See, for instance, Dürr *et al.* (1992): they reconstruct in the pilot-wave theory the guidance equation for the particles (the simplest ontology) postulating a first-order equation (the simplest type of equation), and using symmetries (rotational invariance, time-reversal invariance, and Galilei invariance) to select among the possible otherwise adequate alternatives.

[45] See Allori *et al.* (2008) for an explicit claim about this.

[46] While I think more work needs to be done to clarify the relation between laws and fundamental entities, especially in contrast with grounding and similar relations, the point of the present discussion was to emphasize how by claiming that something belongs to the ontology does not necessarily imply that it represents some material entity, as it instead could represent a (part of the) law.

[47] Maudlin (2019), p. 89.

points, this attitudes towards the quantum state makes the requirement of local beables unnecessarily mysterious. Localists require local beables because, being in three-dimensional space, they connect directly with experience. So, why in the case of the quantum state it does not matter if it is in configuration space? How does it connect with experience? Presumably it is connected to the way in which it 'acts' on the local beables. But if so, what is the difference between this view and the nomological approach? At least the latter tries to provide an explanation of why the wavefunction acts on the local beables (it is part of the law), without merely postulating that it does.

The importance of explanation is also connected with the requirement of familiarity for the beables. Both quantum state primitivists and primitive ontologists claim that they do not want to depart from common sense. But why? The primitive ontologists want to have the simplest possible explanation. Instead quantum state primitivists seem to hold a double standard. First, they claim (and the primitive ontologists agree) that a reason to postulate local beables is that there should be direct connection between theory and experiments. Then they argue (and the primitive ontologists agree) that spacetime state realism (discussed in section 2) is unsatisfactory because even if local the density matrix is not separable, and "this inversion of the usual relation between spatial parts and wholes means that we cannot infer the macroscopic situation […] from the state of the microscopic terms."[48] Thus, if reduced density matrices are the local beables, then we cannot use familiar notions to recover observations from the theory (because they invert the usual part–whole relation). In other words, familiarity arguments have been put forward by quantum state primitivists as well as primitive ontologists to argue for a three-dimensional ontology and against spacetime state realism: if you do not postulate a three-dimensional, separable ontology then you cannot use familiar notions to recover observations from the theory. However, quantum state primitivists *also* want to be able to say that there is no problem that the quantum state, their nonlocal beable, is not familiar.[49] This is difficult to defend: if the quantum state is a primitive nonlocal beable, then we cannot use familiar notions to recover observations from the theory. This argument instead does not affect the primitive ontology approach, as the wavefunction is nomological.

Why do quantum state primitivists want to say that the quantum state is a beable, even if it is nonlocal? Presumably for two reasons. First, the idea that if something appears in the fundamental equations then it corresponds to something in the world *as a material entity*.[50] However, the equations of evolution of the local beables can be written in terms of very different variables. In orthodox quantum mechanics one can use density matrices instead of wavefunctions. Different reformulations of the pilot-wave theory have been proposed, in terms a 'collapsing' wavefunction,[51] density matrices,[52] an infinite set of three-dimensional fields,[53] a multi-field,[54] and so on. One can rewrite the spontaneous localization theory with the density matrix instead of the wavefunction or propose a different equation for the wavefunction.[55] Which of these equations is supposed to be taken as 'fundamental'? Which of these equations is to be used to reify, or materialize, its variables? Primitive ontologists think that none of them is, and call these reformulations *physically equivalent* theories: in orthodox quantum theory the Schrödinger equation and

[48] Maudlin (2019), p. 202.
[49] Maudlin (2019), chapter 3.
[50] Mauldin (2018).
[51] Allori *et al*. (2008).
[52] Allori *et al*. (2014).
[53] Norsen (2010).
[54] Forrest (1988), Belot (2012), Hubert and Romano (2018).
[55] Allori *et al.* (2008).

the wavefunction have been used as a matter of convenience. One could resist the conclusion and assert that these theories are metaphysically inequivalent. Indeed, they provide different pictures of the world: one in which there is the quantum state, one in which there are infinitely many local fields, one in which there is a three-dimensional multi-field, and so on. However, the experimental data are reproduced by the behavior of the local beables alone. The wavefunction, the density matrix, the multi-fields, the multitude of fields, and so on are relevant only insofar as they 'make' the local beable reproduce the data correctly. So, even fixing the local beable, the object which makes the local beable behave in an empirically adequate way is underdetermined. Therefore, it seems to me that the most straightforward attitude towards these entities, including the wavefunction, is to think of them not as representing some unfamiliar, mysterious nonlocal beable, but as part of the laws. This is what the primitive ontologists should argue, I believe, because if one does that, as we have seen, one provides a constructive explanation. Moreover, as we have seen, saying that the wavefunction is part of the law is not to deny it is part of the ontology: the wavefunction exists in the world, without being material, being part of the nomology. Be that as it may, the second reason for quantum state primitivists to think of the wavefunction as a nonlocal beable is that it encodes nonlocality, which is a fact of nature.[56] Granting this, however, does not mean granting the wavefunction has to correspond to some material but mysterious 'thing' in the world. Nonlocality may well be specified by the nomological facts. That is, assuming nonlocality is a fact of nature, there could be several ways of incorporating this in our account of the world. Either assume that there is a quantum state, encoding this nonlocality, or assuming nonlocal laws, which the wave function is part of. I think that the first approach is less plausible than the second, as it leaves many unanswered questions, including: where is the quantum state? How does it interact with 'regular' matter? Why doesn't 'regular' matter act back?

4.2. Explanation and Theory Architecture

The discussion above helps with the distinction between local beables and primitive ontology only up to a point because it is merely about the nature of the wavefunction. The emphasis on explanation helps explain why primitive ontologists find quantum state primitivism unsatisfactory, but it does not clarify the disagreement with other localists (such as Norsen, who does not speculate about the nature of the wavefunction, and the multi-fielders) that certain local ontologies proposed by the localists may not be suitable primitive ontologies. To see the origin of the disagreement we need to move to a theory like classical electrodynamics in which there are local beables, namely the electromagnetic fields, which, I argue, are not part of the primitive ontology. In this subsection, I explore how a theory explains the phenomena depending on which beable one includes in the primitive ontology. The example of classical electrodynamics is instructive not because it provides the most persuasive motivation to think of the electromagnetic fields as not in the primitive ontology. In fact, it does not: as we will see in the next subsection, the most compelling reason that disqualifies the electromagnetic fields as primitive ontology has to do with symmetries. Instead, as we will see, I think the example is interesting because the differences between classical electrodynamics and quantum mechanics help clarifying the reasons why one is inclined to think of a given mathematical object as representing something material.

Classical electrodynamics was built on classical mechanics to account of new experimental data. New types of interactions between particles were postulated, electric and magnetic, which were conveniently described in terms of electric and magnetic fields which extended to every point in space. Initially, these

[56] See Bell (1987), Goldstein *et al.* (2011), Maudlin (2014).

fields were treated as efficient bookkeeping devices rather than part of the world. This attitude changed first with Faraday, who first conceived the electromagnetic fields as physically real. Later Maxwell showed that electromagnetic fields obey a wave-equation with a velocity of propagation equal to the velocity of light, suggesting that light is the vibration of these fields. Moreover, only if we assign energy to these fields, the law of conservation of energy holds for the complete system of particles and fields. So, physicists started to take seriously the idea of a dual physical ontology: matter, whose nature is to be made of particles, and light, which is a vibration of the electromagnetic fields. In this way, one would also keep the conservation of energy. In classical mechanics the world was made of particles, macroscopic objects were composed of them, and the explanation of their macroscopic properties was constructive and dynamical: for instance, the temperature of a body is the mean kinetic energy of the particles constituting it. If there are particles and fields, then the situation slightly changes. The explanation of macroscopic properties is still constructive and dynamical, even if there are also interactions between particles and fields. The difference is mainly that it does not make sense to think that fields are microscopic, as they extend to infinity. Nonetheless there is a clear sense in which the properties of light (diffraction and interference, for instance) are explained dynamically by the wave-equation the fields obey to.[57] Having said this, there is an interesting asymmetry between the electromagnetic fields and the particles: while the particles can generate the fields, the converse is not true. Why? One possibility is to think that fields are somewhat 'derivative,' rather than primitive. Or, without introducing a new and unnecessary terminology, one think that only the primitive beables as physical, dropping the 'derivative' ones out of the primitive ontology. So, particles can generate fields and not the other way around because fields are not primitive like particles. An independent indication that they are not primitive is that they are not at the same level of the architecture of the theory as particles are: they are not at the foundation because they were introduced to account for the particles' behavior as mediators of the interaction between the particles. This is the sense in which they are 'derivative:' there is no force, and thus no electromagnetic field, if there are no particles. Because of this, one may naturally think of them as nomological. So the primitive ontologist suggestion is to resist doing what it was historically done, namely to think of the fields are part of the furniture of the physical world: they are part of the ontology but they belong to the nomology rather than the primitive ontology.

However, there are many difficulties with taking electromagnetic fields as nomological. One problem is that they seem to give the wrong direction in the nomological arrow. A law (in a non-Humean sense) governs the behavior of matter, and thus it makes sense to think of the fields as governing the behavior of the particles. Nonetheless, we just said that the fields are generated by the particles, and this seems to imply that the laws are generated by the particles, which is exactly the opposite of what we want. This tension is mitigated in a Humean picture laws, where the governing metaphor does not hold so strictly. So, one could say that it is *as if* there are fields that sometimes are generated by the particles. Or, from a better perspective, one could be less strict in the interpretation of the fields being are generated by the particles and think that they are 'derivative' of the particles in another sense, rather than created by them. In other words, one could think that God created both particles and the laws, and in creating the latter he

[57] One should distinguish between the case of a dual ontology of particles and fields, and an ontology exclusively of fields. In the latter, since the fields spread out, one would have to reconstruct the definite tracks one finds in detectors presumably in terms of wave-packets localizing macroscopically (as attempted by Schrödinger, when he originally tied to interpret the wavefunction as material). In the case of a dual ontology of particles and fields, the fields instead are seen as the mediators of the force, and thus have a value everywhere, and this is the reason why it is difficult to see them as microscopic. They are more similar to the gravitational potential in classical Newtonian gravity.

created the fields which depend on the particles. Thinking in these terms avoids the problem mentioned above but there are other issues. First, as we have seen, we would have to reject the idea that energy is conserved. While this is possible, it is nonetheless hard to swallow. Moreover, one would have trouble accommodating free fields, those that exists without particles, because fields are not material. While that may be accepted, it would make the nature of light mysterious. One could propose that light does not exist, and that light phenomena can be accounted for in terms of particle phenomena with 'unusual' laws.[58] Again, it is still possible but extremely radical, and it is questionable that these costs are worthwhile. In other words, there are serious difficulties in thinking of the fields as nomological, which may be taken to suggest that fields should be regarded as physical after all. However, I think there is another reason, which I take to be the most compelling one, to reject the fields as physical. As we will see in the next section, the main problem is that if we think of fields are physical, then we lose symmetries, and this is a significant cost. Before we come to that, let me notice how, interestingly, the above-mentioned difficulties in treating electromagnetic fields as nomological do not arise in quantum theory when deciding how to classify the wavefunction. Moreover, one can also invoke other factors to make the case for the nomological wavefunction more straightforward, when compared to electromagnetic fields. In fact:

1) *Configuration space*: the wavefunction, unlike the electromagnetic fields, is not in three-dimensional space.
2) *Right kind of asymmetry*: like electromagnetic fields, the wavefunction acts on the primitive ontology but, unlike them, the primitive ontology does not interact with it.[59]
3) *There are always 'free solutions:'* unlike electromagnetic fields, the primitive ontology can never be thought as generating the wavefunction.
4) *Energy conservation*: the wavefunction has no associated energy, unlike electromagnetic fields, so there is no problem of violation of energy conservation.
5) *There is no 'wavefunction-physical entity:'* unlike electromagnetic fields which constitute light, the wavefunction does not constitute anything.

In other words, all the motivations we had in classical electromagnetism to treat the fields as physical do not hold in quantum theory, and moreover we can think of more reasons to consider the wavefunction nomological. So, the case for the wavefunction to be part of the law is more compelling than the case for the fields, even if we do not add symmetry considerations. What happens if instead we do?

4.3. Symmetries

The primitive ontology approach urges to maximize the number of symmetries of the theory. In this section I argue that if one keeps all the local beables in the primitive ontology then the theory loses symmetries, because this amounts to ignore the architecture of the theory. This is the reason why the example of electromagnetism is important: it shows us how the role of a variable in the theory determines its place in the theory architecture, and its transformation properties under a symmetry.

To see this, take time–reversal symmetry in classical electrodynamics. A possible history of the world consists of a series of instantaneous snapshots capturing what exists at each moment, just like the sequences of frames in a movie. The time-reversed history is one in which the order of the snapshots is

[58] See Lazarovici (2018) for an argument against a field ontology.

[59] Incidentally, the primitive ontology approach solves the problem of back reaction in the pilot-wave theory: why the wavefunction acts on the particles but not the other way around? Because the wavefunction is not material.

reversed, as a movie projected backwards. A theory is invariant under time-reversal if both the forward and the backward stories are possible ways the world can be. Albert, which was the first to point out these issues, notices that physical fields should transform as their nature prescribes.[60] So, what is the nature of electromagnetic fields? Assuming they are represented by vector functions, one should expect that both the electric field E and the magnetic field B would transform under time-reversal by not changing anything but their temporal direction: $E(t) \to E(-t)$ and $B(t) \to B(-t)$. However, with this transformation Maxwell's equations do not yield solutions, thus they do not describe possible ways the world could be. Instead, the transformation should be: $E(t) \to E(-t)$ and $B(t) \to -B(-t)$. Albert argues that there is no reason for B to change like that, and concludes that classical electrodynamics is not time-reversal invariant. *Contra* Albert people have argued that there is a reason for such transformation.[61] The idea is that there is a natural way for a given mathematical object to change under a particular transformation, which depends on its geometrical definition: B is an axial vector, in contrast with E which is a polar vector, and that is why it flips under time-reversal, while E does not. Or E and B are part of the electromagnetic tensor, which is suitably defined to transform such as to make the theory invariant. However, I think that even if we can find which mathematical object makes the transformation needed for invariance natural, still it is hard to swallow that the fields are part of the ontology. In fact, what is depicted in a time-slice has to change depending on whether it is taken from the forward or the backward movie: while a snapshot extracted at time t from the forward movies shows the electromagnetic fields being E and B, the snapshot extracted from the backward movie at the same instant shows instead E and $-B$. It is as if a snapshot from *The Empire Strikes Back* projected forward depicts a green and short Yoda, while the snapshot corresponding to very same instant but taken from the backward movie shows a giant blue alien instead. This is incomprehensible: if the electromagnetic fields are thought of as physical, regardless of whether we think of them as part of the electromagnetic tensor or not, their intensity should be the same, and they should point in the same direction regardless of whether the snapshot which portraits them is taken from the backward or forward story. Instead this is exactly what happens if we take them to be physical, and we think of the theory as invariant.[62] So we only have two other options. First, as Albert maintains, the fields are physical, but they need to be thought of as independent from one another, which is implausible, and the theory is no longer invariant under time-reversal. Otherwise, we think of the fields as part of the nomology, as the primitive ontologists suggest.

Now let's carry over this example to quantum theory. The theory is invariant under time-reversal if the wavefunction becomes its complex conjugate: $\psi(t) \longrightarrow \psi^*(-t)$. However, if the wavefunction were a material, or physical, field in configuration space as the wavefunction realists maintain, there would be no reason why it would transform like that, as it would merely flip time direction.[63] As in classical electrodynamics, some authors have maintained that the reason why the wavefunction transforms in this

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[60] See Albert (1996).

[61] See Arntzenius (2004), Earman (2002), Malament (2004). For more on this, see North (2008), Allori (2015 c) and references therein.

[62] See Allori (2019 a).

[63] The same is true for other symmetries, like Galilei invariance: under a Galilei transformation $x \to x - vt$ the wavefunction being a filed would transform as: $\psi(x) \longrightarrow \psi(x - vt)$. Namely, it would be boosted in the direction of the velocity, but its intensity would not change. However, such transformation would not leave the theory Galilei invariant: it would not map solutions into solutions, as a more complicated transformation involving an exponential would be needed (Allori 2018). But what would justify a transformation like this?

way is because it is a projective ray, and not a field in configuration space.[64] Wavefunctions that differ by a multiplication of a complex number of unitary norm are called projective rays in a projective Hilbert state $P(H)$ of Hilbert space H, the space of the totality of the wavefunctions. The ray is thus the set of equivalence classes of elements of H that differ by a non-zero complex number c, like ψ and $c\psi$. An important reason to think the wavefunction is a ray is that wavefunctions which differ from a non-zero complex number generate the same transition probability, and therefore they can be taken as representing the same state of affairs.[65] If the wavefunction is a ray, then one can show that it is in the nature of rays to transform as needed so the theory is invariant. In this way, by assuming that the operator which implements the transformation connected with the invariance acts on an object as the nature of the object prescribes,[66] one can explain why the wavefunction transforms in the way it does to preserve invariance.

However, as emphasized above, an important problem remains, and this constitutes a general argument against the materiality of the wavefunction. As in electrodynamics, in order for the backward story to be empirically adequate, what is depicted in a single snapshot has to change depending on whether it is taken from the forward or the backward movie: while a snapshot extracted at time t from the forward movies shows the wavefunction ψ, the snapshot extracted from the backward movie at the same instant has to show ψ^*. Again, however, the entities in the ontology depicted in the time-forward and the time-backward histories should merely change in their temporal direction: if Yoda has a given color and a given height, then Yoda is green and short, regardless of whether we watch *The Empire Strikes Back* forward or backwards.

So, I think that Albert is correct in thinking that there is no reason for the wavefunction to transform that way if it is a physical field. But I also think that considering the wavefunction as a ray is problematical as well because, regardless of whether there is a justification of why the wavefunction transforms as it must to preserve invariance, if one assumes that the wavefunction is material, this implies that the material ontology changes depending on whether one looks at the world from the forward or the backward point of view. This shows, in my opinion, that the quantum state, ray or field, cannot be material. Otherwise, the only option is that the theory would not be invariant.

Norsen and the multi-fielders wish to think of the wavefunction as a local beable, similar to an electromagnetic field. However, the analysis above shows that the wavefunction cannot be thought as representing something material, as there is no reason why the ontology it represents should change in absurd ways under symmetry transformations. This leaves us to think of the wavefunction as not material: in this sense, even if one could construct a theory in which the wavefunction is reduced to one of the other local beables, as Norsen or the multi-fields would like, the local beable would still not be a good primitive ontology because the theory would lose symmetry properties. In fact, these attempts amount to ignore the fact that variables have a distinctive role in the theory: the fields in electrodynamics, and the wavefunction in quantum theory, are not primitive, they do not represent matter but rather they represent the interaction, the law of evolution for the variables at the foundations. Ignoring this, either destroys symmetries, or makes the ontology behaving incomprehensibly, as in the Yoda example. Instead, thinking of the wavefunction (or the electromagnetic fields) as nomological explains why the

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[64]See Earmann (2002) and Roberts (2017) for time-reversal symmetry, see Allori (2018) for Galilei invariance. See also Roberts (2019) and references therein, for further discussion.
[65] See Wigner (1939), Shankar (1994).
[66] See Skow (2010).

wavefunction is a ray (or the fields are part of the electromagnetic tensor), namely because its role is to generate the evolution of the primitive ontology so that the theory is invariant.

In the primitive ontology framework, the wavefunction is not part of the primitive ontology (matter), and the law which needs to be invariant is the one for the primitive ontology. Since this approach assumes that there is nothing in the material world which correspond to the quantum state, then one can forget about justifying why the wavefunction behaves in that peculiar way which preserves the symmetry. I have previously argued that the best way of thinking of the wavefunction is as a projective ray in Hilbert space, because it is in the nature of projective rays to transform in the way which happens to preserve Galilei invariance.[67] However, there is an important sense in which this justification was not necessary: the quantum state does not exist in physical space, the wavefunction is just one, among the possible ones, convenient mathematical representation of the law which describes the evolution of the wavefunction. As such, it can do whatever it is needed to preserve the symmetry of the theory.

5. Conclusion

In this paper I have explored the differences between two approaches which I think have not been sufficiently distinguished: one based on the notion of local beable and another based on the notion of primitive ontology. I have argued that the primitive ontology approach is stricter because the primitive ontology not only 1) needs to represent matter in *three-dimensional space* (like required by local beables approaches), but also 2) this approach is committed to a *dynamical, constructive explanatory schema*, which 3) *preserves symmetry properties* of the theory. Because of these reasons, certain local beables, such as electromagnetic fields, would not be suitable primitive ontologies. Regardless of whether one has been persuaded by my arguments that one should prefer the primitive ontology approach to other localist approaches, I hope that this paper helped clarifying that these approaches are distinctively different.

References

- Albert, D.Z. 1996. "Elementary Quantum Metaphysics". In J.T. Cushing, A., Fine, S. Goldstein (eds.) *Bohmian Mechanics and Quantum Theory: an Appraisal:* 277–284. Netherlands: Springer.
- Albert, D.Z. 2015. *After physics*. Cambridge: Harvard University Press.
- Albert, D.Z. & A. Ney 2013. *The Wave Function: Essays in the Metaphysics of Quantum Mechanics*. New York: Oxford University Press.
- Allori, V., S. Goldstein, R. Tumulka, & N. Zanghì 2014. "Predictions and Primitive Ontology in Quantum Foundations: A Study of Examples." *The British Journal for the Philosophy of Science* 65 (2): 323-352.
- Allori, V., S. Goldstein, R. Tumulka, &N. Zanghì 2011. "Many-Worlds and Schrödinger's First Quantum Theory." *The British Journal for the Philosophy of Science* 62 (1): 1–27.
- Allori, V., S. Goldstein, R. Tumulka, & N. Zanghì 2008. "On the Common Structure of Bohmian Mechanics and the Ghirardi-Rimini-Weber Theory." *The British Journal for the Philosophy of Science* 59 (3): 353-389.
- Allori, V. manuscript a. "Modified Schrödinger Dynamics and the Incompatibility of Quantum Theory with Scientific Realism."
- Allori, V. manuscript b. "Wave-functionalism".

[67] Allori (2018).

- Allori, V. 2020 a. "Spontaneous Localization Theories with a Particle Ontology". In V. Allori, A. Bassi, D, Dürr and N. Zanghì (eds.) *Do Wave Functions Jump? Perspectives on the Work of GianCarlo Ghirardi*. Springer.
- Allori, V. 2020 b. "Why Scientific Realists Should Reject the Second Dogma of Quantum Mechanics." In: M. Hemmo and O. Shenker (eds.) *Quantum, Probability, Logic: the Work and Inlfuence of Itamar Pitowsky*: 19-48. Springer.
- Allori, V. 2019 a. "Quantum Mechanics, Time and Ontology." *Studies in History and Philosophy of Modern Physics* 66: 145-154.
- Allori, V. 2019 b. "Scientific Realism without the Wave-Function." In: J. Saatsi, S. French (eds.) *Scientific Realism and the Quantum*: 212-228. Oxford: Oxford University Press.
- Allori, V. 2018. "A New Argument for the Nomological Interpretation of the Wave Function: The Galilean Group and the Classical Limit of Nonrelativistic Quantum Mechanics." *International Studies in the Philosophy of Science* 31 (2): 177-188.
- Allori V. 2016. "Primitive Ontology and the Classical World." In: R. Kastner, J. Jeknic-Dugic, G. Jaroszkiewicz (eds.), *Quantum Structural Studies: Classical Emergence from the Quantum Level:* 175-199. World Scientific.
- Allori V. 2015 a. "Primitive Ontology in a Nutshell." *International Journal of Quantum Foundations* 1 (3): 107-122.
- Allori V. 2015 b. "Quantum Mechanics and Paradigm Shifts." *Topoi* 32 (2): 313-323.
- Allori V. 2015 c. "Maxwell's Paradox: Classical Electrodynamics and its Time Reversal Invariance." *Analytica* 1:1-19 (2015).
- Allori V. 2013a. "Primitive Ontology and the Structure of Fundamental Physical Theories." In: D. Albert, A. Ney (eds.) *The Wave Function: Essays in the Metaphysics of Quantum Mechanics*: 58-75. New York: Oxford University Press.
- Allori V. 2013b. "On the Metaphysics of Quantum Mechanics." In: S. Lebihan (ed.), *Precis de la Philosophie de la Physique:* 116-151. Vuibert.
- Arntzenius, F. 2004. "Time-reversal Operation, Representation of the Lorentz Group and the Direction of Time." *Studies in Histories and Philosophy of Modern Physics* 35(1): 31-43.
- Bacciagaluppi, G. & A. Valentini 2009: *Quantum Theory at the Crossroads: Reconsidering the 1927 Solvay Conference*: 341-371, Cambridge University Press.
- Barrett, J. 1999. *The Quantum Mechanics of Minds and Worlds*. New York: Oxford University Press.
- Bell, J.S. 2004. *Speakable and Unspeakable in Quantum Mechanics.* Cambridge University Press.
- Bell, J. S. 1987. "Are there Quantum Jumps?" In C. W. Kilmister (ed.) *Schrödinger: Centenary of a Polymath*, Cambridge University Press. Reprinted in Bell, J. S. 2004: 201-212.
- Beller, M. 1999. *The Quantum Dialogue: The Making of a Revolutions*. Chicago: University of Chicago Press.
- Belot, G. 2012. "Quantum States for Primitive Ontologists." *European Journal for Philosophy of Science* 2(1), 67–83.
- Bhogal, H. & Z. Perry. 2017. "What the Humean Should Say about Entanglement." *Noûs* 51 (1): 74–94.
- Bloch, F. 1976. "Heisenberg and the Early Days of Quantum Mechanics." *Physics Today.*
- Brown, H.R. & C. Timpson. 2006. "Why Special Relativity Should Not Be a Template for a Fundamental Reformulation of Quantum Mechanics." In W. Demopoulos, I. Pitowsky (eds.) *Physical Theory and its Interpretation: Essays in Honor of Jeffrey Bub*: 29-42. Springer.
- Callender, C. 2015. "One World, One Beable." *Synthese* 192: 3153.
- Chen, E. K. 2017. "Our Fundamental Physical Space: An Essay on the Metaphysics of the Wave Function." *Journal of Philosophy* 114 (7): 333-365.

- Cushing, J. T. 1994. *Quantum Mechanics: Historical Contingency and the Copenhagen Hegemony.* Chicago: University of Chicago Press.
- Dürr, D., S. Goldstein, & N. Zanghì, 1992. "Quantum Equilibrium and the Origin of Absolute Uncertainty." *Journal of Statistical Physics* 67: 843-907.
- Earman, J. 2002. "What Time-reversal Invariance is and Why it Matters." *International Studies in the Philosophy of Science* 16: 245-264.
- Emery, N. 2017. "**Against Radical Quantum Ontologies."** *Philosophy and Phenomenological Research.* 95(3): 564–591.
- Einstein, A. 1982. *Ideas and Opinions*. New York: Crown Publishers, Inc.
- Einstein, A. 1926. Einstein to Paul Ehrenfest, 18 June, 1926, EA 10-138. Translated in Howard (1990), p. 83.
- Einstein, A. 1919. *What is the Theory of Relativity?* The London Times. Reprinted in Einstein, A. 198.: 227–32.
- Esfeld, M., D. Lazarovici, M. Hubert & D. Dürr 2014. "The Ontology of Bohmian Mechanics." *The British Journal for the Philosophy of Science* 65 (4): 773-796.
- Esfeld. M. 2014a. "The Primitive Ontology of Quantum Physics: Guidelines for an Assessment of the Proposals." *Studies in History and Philosophy of Modern Physics* 47: 99-106.
- Esfeld, M. 2014b. "Quantum Humeanism, or: Physicalism without Properties." *The Philosophical Quarterly* 64: 453–470.
- Fine, A. 1996. *The Shaky Game: Einstein Realism and the Quantum Theory*. Chicago: University of Chicago Press.
- Forrest, P. 1988. *Quantum metaphysics*. Oxford: Basil Blackwell.
- Ghirardi, G. C., R. Grassi, & F. Benatti 1995. "Describing the Macroscopic World: Closing the Circle within the Dynamical Reduction Program." *Foundations of Physics* 25, 5–38.
- Goldstein, S., T. Norsen, D. Tausk, & N. Zanghì, 2011. "Bell's Theorem." *Scholarpedia*, 6(10): 8378.
- Goldstein, S. & N. Zanghì 2013. "Reality and the Role of the Wavefunction. In Quantum Theory." In D. Z. Albert and A. Ney (eds.) *The Wave-function: Essays on the Metaphysics of Quantum Mechanics*: 91-109. New York: Oxford University Press.
- Goldstein, S. 1998. "Quantum Theory without Observers. Part One." *Physics Today*, March: 42-46. "Part Two." Physics Today, April: 38-42.
- Healey, R. 2002. "Can Physics Coherently Deny the Reality of Time?" In C. Callender (ed.) *Time, Reality and Experience*: 293–316. Cambridge: Cambridge University Press.
- Howard. D. 1990. "'Nicht Sein KannWas Nicht Sein Darf,' or the Prehistory of EPR, 1909-1935: Einstein's Early Worries about the Quantum Mechanics of Composite Systems." In A.I. Miller (ed.) *Sixty-Two Years of Uncertainty.* New York: Plenum Press.
- Hubert, M. & D. Romano 2018. "The Wave-function as a Multi-field." *European Journal for Philosophy of Science* 8: 521–537.
- Huggett, N., & C. Wüthrich 2013. "Emergent Spacetime and Empirical (in) coherence". *Studies in Histories and Philosophy of Science* B44: 276-285
- Lazarovici, D. 2018. "Against Fields." *European Journal for Philosophy of Science* 8(2): 145-170.
- Lewis, P. J. 2016. *Quantum Ontology: A Guide to the Metaphysics of Quantum Mechanics*. Oxford: Oxford University Press.
- Malament, D. 2004. "On the Time-reversal Invariance of Classical Electromagnetic Theory." *Studies in Histories and Philosophy of Modern Physics* B 35: 295-315.
- Maudlin, T. 2019. *Philosophy of Physics, Quantum Theory*. Princeton: Princeton University Press.
- Maudlin, T. 2018. "Ontological Clarity via Canonical Presentation: Electromagnetism and the Aharonov–Bohm Effect." *Entropy* 20(6): 465.
- Maudlin T. 2014. "What Bell Did." *Journal of Physics A: Mathematical and Theoretical* 47: 424010.

- Maudlin. T. 2013: “The Nature of the Quantum State.” In D.Z. Albert and A. Ney (eds.) *The Wave Function: Essays in the Metaphysics of Quantum Mechanics*: 126-153. New York: Oxford University Press.
- Maudlin, T. 2007. “Completeness, Supervenience, and Ontology.” *Journal of Physics* 40: 3151-3171.
- Miller, E. 2014. “Quantum Entanglement, Bohmian Mechanics, and Humean Supervenience.” *Australasian Journal of Philosophy* 92(3): 567–83.
- Monton, B. 2002: “Wave-function Ontology.” *Synthese* 130: 265-277.
- Monton, B. 2006: “Quantum Mechanics and 3N-Dimensional Space.” *Philosophy of Science* 73 (5): 778-789.
- Monton, B. 2013: “Against 3-N-Dimensional Space.” In D. Z. Albert and A. Ney (eds.) *The Wave-function: Essays in the Metaphysics of Quantum Mechanics*: 154- 167. New York: Oxford University Press.
- Myrvold, W. 2017. “Ontology for Collapse Theories.” In S. Gao (ed.) *Collapse of the Wave Function*. Cambridge: Cambridge University Press.
- Myrvold, W. 2015. “What is a Wave Function?” *Synthese* 192(10): 3247-3274.
- Ney, A. forthcoming. *Finding the World in the Wavefunction.* Oxford University Press.
- Ney, A. 2019. “Locality and Wave Function Realism.” In O. Lombardi, S. Fortin, C. Lopez, F. Holik (eds.) *Quantum Worlds, Perspectives on the Ontology of Quantum Mechanics: 164-182.* Cambridge University Press.
- Ney, A. 2015. “Fundamental Physical Ontologies and the Constraint of Empirical Coherence.” *Synthese* 192: 3105–3124.
- Ney, A. 2012. “The Status of Our Ordinary Three Dimensions in a Quantum Universe.” *Nous* 46(3); 525-60.
- North, J. 2013. “The Structure of the Quantum World.” In Albert, David Z., and Ney, Alyssa (eds.) *The Wave-function: Essays in the Metaphysics of Quantum Mechanics*: 184–202. New York: Oxford University Press.
- Norsen, T., D. Marian & X. Oriols, 2015. “Can the Wave Function in Configuration Space be Replaced by Single-particle Wave Functions in Physical Space?” *Synthese* 192: 3125–3151.
- Norsen, T. 2010. “The Theory of (Exclusively) Local Beables.” *Foundations of Physics* 40(12): 1858–1884.
- North, J. 2008. “Two Views on Time Reversal.” *Philosophy of Science* 75: 201-223.
- Przibram, K. 1967. *Letters on Wave Mechanics.* Martin Klein, trans. Philosophical Library.
- Roberts, B. W. 2017. “Three Myths about Time Reversal in Quantum Theory.” *Philosophy of Science* 84 (2): 315-334.
- Roberts B.W. 2019. “Time Reversal.” In E. Knox and A. Wilson (eds.) *The Routledge Companion to the Philosophy of Physics*. Routledge.
- Romano, D. 2020. “Multi-field and Bohm’s Theory”. *Synthese* (2020). https://doi.org/10.1007/s11229-020-02737-6
- Schrödinger, E. 1926. “Quantisierung als Eigenwertproblem (Zweite Mitteilung).” *Annalen der Physik* 79, 489-527. English translation: “Quantisation as a Problem of Proper Values. Part II”.
- Schrödinger, E. 1935. Schrödinger to Einstein, 19 August 1935. translated in Fine (1996), p.82.
- Shankar, R. 1994. *Principles of Quantum Mechanics*. New York: Plenum Press.
- Skow, B. 2010. “On a Symmetry Argument for the Guidance Equation in Bohmian Mechanics.” *International Studies in the Philosophy of Science* 24 (4): 393-410.
- Spekkens, R.W. 2007. “In Defense of the Epistemic View of Quantum States: a Toy Theory”. *Physical Review Letters* A75: 032110.
- Suárez, M. 2105: “Bohmian Dispositions.” *Synthese* 192(10):3203–28.
- Wallace, D. & C. Timpson 2010. “Quantum Mechanics on Spacetime I: Spacetime State Realism”. *The British Journal for the Philosophy of Science* 61(4):697–727.
- Wallace, D. 2020. “Against Wave Function Realism”. In S. Dasgupta and B. Weslake, (eds.) *Current Controversies in Philosophy of Science*: 63-74. London: Routledge.

- Wigner, E. P. 1939. "On the Unitary Representations of the Inhomogeneous Lorentz Group." *Annals of Mathematics* 40: 149–204.
- Tumulka R. 2016. "Paradoxes and Primitive Ontology in Collapse Theories of Quantum Mechanics". In S. Gao (ed.) *Collapse of the Wave Function: Models, Ontology, Origin, and Implications:* 134-153. Cambridge University Press.