Against the Measurement Problem: On the Incompatibility between Scientific Realism and Quantum Mechanics

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Abstract

The measurement problem is traditionally considered *the* problem to solve in order to restore the compatibility between quantum theory and scientific realism. In this paper instead I argue that it is not straightforward to spell out what the incompatibility problem actually is, and that different types of realists will think of it in different ways. First, I distinguish between a *robust* version of realism, which looks for a fundamental description of reality, and a *relaxed* version, which looks for a description of the regularities in the phenomena. I argue that while the relaxed realist will naturally think of the measurement problem as a problem of precision, the robust realist will think of it as a completeness problem. I also maintain that each kind of realism comes with a natural explanatory structure: while the robust realist will find satisfactory constructive theories, in which the phenomena are dynamically explained, the relaxed realist will be happy with principle theories, which provide constraints on them. In this regard, I show that the spontaneous localization theory, thought as a theory about the wavefunction (dubbed bare GRW), is a non-constructive dynamical hybrid. This creates two tensions for the wavefunction realist endorsing bare GRW. First, they seem relaxed realists in denial, as bare GRW's explanation is not constructive. This leaves them with relaxed realism, which however is arguably too weak to be truly realism by their standards. In addition, there seems to be an explanatory mismatch between the non-constructive quantum explanation, and the constructive derivation of thermodynamics from the microscopic dynamics which appears to be problematic for a realist of any kind.

Keywords: measurement problem; completeness problem; spontaneous localization theory; scientific realism; wavefunction realism; principle and constructive theories.

1. Introduction

Quantum theory is one of the greatest accomplishment of scientific inquiry as it has an outstanding predictive power, both in depth and in breath: allegedly, with a single mathematical object, the wavefunction, and a simple enough equation, the Schrödinger equation, the theory delivers predictions which are incredibly accurate, and accounts for a vast variety of physical phenomena. However, people have developed quantum theories which modify the Schrödinger equations. So the natural question to ask is *why such theories were ever proposed*: haven't we just noticed that the theory is empirically adequate? The traditional answer to this question is that quantum theory faces a challenge at the interpretational level, the so-called measurement problem. In other words scientific realists, who think that our best theories inform us about the nature of reality, think that quantum theory is incompatible with realism. However, they also think that one can make quantum theory amenable to a realist interpretation by solving the measurement problem. To put it differently, if one dubs the problem of incompatibility of quantum theory with scientific realism *the realism problem*, then

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traditionally scientific realists have maintained that *the measurement problem is the realism problem*. The measurement problem arises by noticing that the assumption that the description of a physical system provided by the wavefunction is complete, and the fact that the wavefunction evolves according to the Schrödinger equation cannot be maintained at the same time ("Either the wavefunction, as given by the Schrodinger equation, is not everything, or is not right").[2] If we allow them both to be true we end up with macroscopic superpositions, such as the instrument displaying all possible measurement outcomes at the same time. Solutions of the measurement problem are found by denying each assumption, or by embracing the conclusion. So, one way of denying that the wavefunction always evolves according to the Schrödinger equation is accomplished by postulating the collapse rule introduced by John von Neumann:[3] the wavefunction evolves according to the Schrödinger equation until a measurement is performed, and then it collapses into one of the terms of the superposition. Another, more satisfactory, way of doing the same thing is provided by the spontaneous localization theory of GianCarlo Ghirardi, Alberto Rimini and Tullio Weber[4] which makes precise 'when' and 'where' the wavefunction collapses without referring to measurement processes. Alternatively, one could reject the claim that the wavefunction provides a complete description, which is the route taken by the pilot-wave theory: one also needs to specify the particles' position.[5] The last option is to accept that measurements have no definite results, as defended by the many-worlds theory: 'whatever can possibly happen, will indeed happen,' not here but in another 'world.'[6]

Usually these three solutions above are taken to provide equally satisfactory solutions of the measurement problem as the realism problem. That is, each provides a suitable fundamental understanding of physical reality. Instead, in section 2 I identify different varieties of measurement problem which correspond to different takes on what the realism problem is supposed to be. There is a *problem of empirical adequacy*: the theory cannot be left as is because macroscopic superpositions are not observed. This is fixed by the collapse rule. However, we do not know exactly when to apply this rule, as we do not know what a measurement process actually is, precisely. So, we have a *problem of precision*. In addition, even if we are precise, we have the problem that the object that quantum theory employs, namely the wavefunction, is unlike any other object used in previous theories, as it is not a field in three-dimensional space. This suggested to some that it may not be the right object to describe physical reality, thus producing a *problem of completeness*. In section 3 I show how one can distinguish between different types of scientific realists, each corresponding to a different attitude towards the realism problem. I distinguish between *robust and relaxed realism*: the former being interested in providing a fundamental description of the phenomena, and the latter instead interest in the

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[2] Bell (1987).
[3] von Neumann (1932).
[4] Ghirardi, Rimini, and Weber (1986).
[5] de Broglie (1927).
[6] Everett (1957).

phenomena themselves. I argue that the relaxed realists naturally take the realism problem as either the empirical adequacy or the precision problem, while the robust realists think of it as a completeness problem. As we will see, this implies that a relaxed realist, focusing merely on the phenomena, allows for *more* theories to be satisfactory than the robust realist, who looks instead for a completion of the theory in terms of a more fundamental picture. So any theory which solves the measurement problem understood as the precision problem, including the many-worlds theory and the spontaneous localization theory, will be acceptable for the relaxed realist. However, since these theories solve the problem without supplementing anything to the description provided by the wavefunction, they would not be satisfactory for the robust realist. As I show in section 4, this reading is compatible with the distinction between the so-called *small and big measurement problem* proposed by Itamar Pitowsky,[7] if we identify the former with the precision problem and the latter with the completeness problem. Jeffrey Bub and Pitowsky[8] argue that one can be realist even if solving the small problem. In fact the many-worlds theory is considered a realist interpretation and it only solves the small problem. In my language that means that the many-worlds theory is an example of a relaxed realist theory, rather than a robust one. I argue that the same is true for the spontaneous localization theory, and even for the so-called bare GRW, the spontaneous localization theory according to which the wavefunction represent the fundamental nature of reality. In section 5 I analyze the issue of which kind of realism proponents of this theory seems to endorse in connection with the *principle/constructive theories* distinction proposed by Albert Einstein.[9] Principle theories aim at accounting for the phenomena kinematically, while constructive theories aim at figuring out the reality behind them and explain the phenomena dynamically. I maintain that relaxed realists are likely to find principle theories satisfactory, while robust realists will look for constructive theories. On the basis of this I argue in section 6 that wavefunction realists, who have argued that their framework is particularly suitable in the case of spontaneous localization theory and many-worlds theories, may be thought of *relaxed realists in denial:* even if they think of themselves as robust realists, because they aim at giving us a description of fundamental reality, they however end up endorsing a different, weaker, type of realism than the one they take themselves to support. This raises a consistency challenge for the wavefunction realists as they seem to endorse the same type of realism of approaches they find unsatisfactory. Moreover, I show that bare GRW is a hybrid theory which provides an explanatory schema which is not constructive, but in which the dynamics plays a distinctive role. This may also lead to an explanatory mismatch if the kind of explanation wavefunction realists find satisfactory in other domains is constructive. In fact, some wavefunction realists defend the Boltzmannian explanation of the laws of thermodynamics from the fundamental microscopic laws of classical mechanics, which is a constructive understanding. However, since the explanation wavefunction realism provides in the quantum domain is not constructive, it does not seem

[7] Pitowsky (2007).
[8] Bub and Pitowsky (2010).
[9] Einstein (1919).

consistent on their part to say, as some do, that the Boltzmannian schema does not fundamentally change when one moves from classical to quantum theories.

2. Different Types of Problems …

When discussing the foundations of quantum mechanics, it is always pointed out that quantum theory is unable to provide a microscopic description of the world, and that what it gives us instead is merely a recipe for predicting experimental results. In other words, quantum theory is an instrumentalist theory: its purpose is not to inform us about the nature of reality but rather to reproduce and predict measurement outcomes. As such, it is incompatible with scientific realism, the view that theories can give us information about the nature of the world. Let us call this incompatibility *the realism problem*. Traditionally, it is argued that the realism problem is the measurement problem. That is, in order for quantum theory to describe the world, it needs to be modified to solve the measurement problem. As anticipated and as we will see better in the following, this problem has three possible solutions: either the wavefunction is not complete, or it does not evolve according to the Schrödinger equation, or measurements have no definite results. However, as I show in the rest of the section, there is no unique way of thinking where the realism problem lies, as different people find the problem to be different. So, let us see what these problems are one at the time.

2.1. The Measurement Problem as a Problem of Empirical Adequacy

Let's start from the quantum physics books. There we find an equation, namely the Schrödinger equation, which is an equation about the temporal evolution of an object called the wavefunction, usually denoted with the letter ψ. Assume that a solution of this equation describes possible states of affairs which are observed to happen. The Schrödinger equation is a wave equation (in particular, mathematically, it is linear), so that any sum of solutions of this equation is a solution itself. There is nothing particularly striking or problematic in this, as it is just a regular feature of equations describing waves: waves superimpose with one another (think of how, for instance, water waves in a pond can interfere with one another creating different wave patterns). However, a problem arises when one realizes that if every system is completely described by the wavefunction, and the wavefunction evolves according to the Schrödinger equation, then the superpositions of the wavefunction will be visible at the macroscopic scale. To see this, consider the formalism of quantum theory which can be summarized as follows. Physical systems are described by two mathematical objects: the wavefunction, and self-adjoint operators (operators are just linear functionals, which transform what they act on in something else). It is assumed that self-adjoint operators provide information about properties of objects such as energy, or momentum, as follows. Given a self-adjoint operator A, consider a wavefunction ϕ_i such that the action of A on ϕ_i amounts to the multiplication of ϕ_i by a real number a_i ($A\phi_i = a_i\phi_i$). This wavefunction ϕ_i is called an *eigenstate* of that operator, and the number a_i is called the *eigenvalue* of that eigenstate. So the sense in which operators represent properties is that the eigenvalues of a given operator, say the one associated with energy, represent possible values of the energy, when the system's energy is measured. Having said that, because of the linearity of the Schrödinger equation, the wavefunction of a system can (in particular) be written as a weighted superposition of eigenstates of the operator connected with the property which is being measured: $\psi = \sum_i a_i \phi_i$. Each term of the sum represents a possible measurement result, and the superposition represents all outcomes happening at the same time.

Now here is the measurement problem, first version: [10] if the wavefunction describes everything, and it evolves linearly according to the Schrödinger equation, then measurements will have no definite results. In fact since each measurement outcome is represented by the eigenvalue corresponding to a given eigenstate, and each eigenstate is a solution of the Schrödinger equation, then also the superpositions of all eigenstates is a solution, implying that it is possible to find as a measurement result a superposition of all possible outcomes. However, we never find more than one result for each experiment: the pointer of the apparatus always points in a definite direction. Thus, the theory is not empirically adequate: it does not successfully predict what we directly observe. In this way, *the measurement problem is, first and foremost, a problem of empirical adequacy*, as the theory does not match what we observe. This problem is usually expressed more vividly following Erwin Schrödinger's cat example in his 1935 paper.[11] Assume that there is a cat in a room together with a vial of poison which, if broken, would kill the cat. A mechanism to break the vial is activated by a radioactive decay.[12] Since the complete description of the cat is given by the wavefunction, and the wavefunction can be in superposition of atom-having-decayed and not-having decayed, then it follows that this superposition will spread to the cat which will therefore be in the state of being dead-and-alive at the same time, which is however never the case.

The traditional way in which this problem is addressed in physics books is as formalized by John von Neumann. This amounts to postulating that there are two evolution equations for the wavefunction: the wavefunction evolves according to the Schrödinger equation not all the time, but only until a measurement is performed. When a given property of the system is measured, the wavefunction randomly and instantaneously collapses into a particular eigenstate of the operator representing the measured property, and the result of the measurement is represented by the corresponding eigenvalue.

2.2. The Measurement Problem as a Problem of Precision

However many people find von Neumann's collapse postulate unsatisfactory because it is unclear when one is supposed to apply it (it is unclear what constitutes a measurement), and why it actually applies (it is unclear why a measurement process is not a particular type of physical interaction between the system and the measurement apparatus). In other words, for

[10] See for instance Albert (1992); Maudlin (1995); Myrvold, (2017).

[11] Schrödinger (1935).

[12] For a less cruel example, see Bell (2007).

some *the measurement problem also is a problem of precision*: not only we need to solve the empirical adequacy problem, we also need to know when the rule that solves it is applied, and why it works. That is to say that the adequacy problem shows us we need to eliminate unobserved macroscopic superpositions but we need to eliminate them by using a rule which *does not rely on the notion of observer and measurement*, because observers and measurements are thought to be physical objects are processes like any other. How do we do this? The solutions of the problem of precision are found by observing that the tension is between: a) measurements having definite results (no macroscopic superpositions ever being observed), b) the wavefunction providing the complete description of every physical system, and c) the wavefunction evolving according to the Schrödinger equation. Correspondingly we have three distinct solutions of this precision problem: a) one which grants superpositions a physical significance, b) one which would deny that the wavefunction provides the complete description, and c) one which would deny that the wavefunction evolves according to the Schrödinger equation, without making reference to an observer or a measurement. In this way, there are three distinct approaches to make quantum theory precise: the first route has been taken by the many-worlds theory, in which the different terms of the superpositions exists but never interact with one another so that they are never observed; the second one has been developed in the pilot-wave theory framework in which the complete state of the system is specified by the wavefunction and by the particles' position; while the third possibility has been explored by the spontaneous localization theory. As we will discuss more in section 4.2, in this theory the wavefunction ceases to evolve according to the Schrödinger equation as a matter of law, rather than because of an additional postulate, and the wavefunction spontaneously collapses in one of the terms of the superposition for macroscopic objects, regardless of whether someone performs a measurement on them. The wavefunction, which is all there is in this theory, does not evolve according to the Schrödinger equation. Rather, it evolves according to a nonlinear stochastic equation which includes suitable parameters that makes it the case that the wavefunction corresponding to what we consider macroscopic objects rapidly collapses in a small spatial region after a very short time.

2.3. The Measurement Problem as a Completeness Problem

However, there is another take on what the problem of realism really is. In this view, as I argue in the following, the realism problem is that *quantum theory is fundamentally incomplete*, as it is a theory of the wavefunction, which is not suitable to describe a physical field. In this framework, even if one solves the measurement problem thought as the problem of precision (namely the problem of getting rid, precisely, of the macroscopic superpositions) still this does not necessarily give us a satisfactory theory. In fact, one may solve the precision problem without solving the completeness problem, and solving this problem is considered to be necessary to arrive to a theory compatible with realism. If so, only solving the precision problem in a particular way (namely by completing quantum theory) will solve the realism problem. To better understand this attitude and where it comes from, let me take a short historical detour.

The measurement problem is usually tracked down to Schrödinger's 1935 cat paper, but the fact that quantum mechanics was conceptually problematic was clear much earlier than that. In 1926, around the time of the Solvay conference, people were debating about the meaning of the theory.[13] There were instrumentalist approaches championed by Werner Heisenberg's matrix mechanics, which was a pure mathematical theory to reproduce measurement results without providing any understanding or any intelligible mechanism underlying it. Niels Bohr proposed his wave-particle duality thesis, that microscopic entities have no definite wave or particle nature to show that we do not have adequate concepts to describe the microscopic world.[14] However, there was a lot of resistance, as people attempted realist approaches. For instance Louis de Broglie refused to give in to wave-particle duality and proposed his theory of waves and particles.[15] Originally the theory had a particle equation but lacked a wave equation, which was supplied by Schrödinger in 1926, with his now-called Schrödinger equation. However, Schrödinger got rid of the particles, presumably because he wanted to provide a purely wave account of matter.[16] Be that as it may, Schrödinger's theory was accepted with enthusiasm by many, including notable physicists like Einstein and Hendrik Lorentz because it left some room for a realist understating of the theory. Nonetheless Lorentz complained that he would be obliged to go back to matrix mechanics if Schrödinger could not solve what he thought was a fundamental problem. For a many-particle system the wavefunction is not in three-dimensional space but it is in configuration space (called below q-space), namely the space of configurations of all particles. By definition, since each particle has three-dimensions, assuming there are N particles in the world, configuration space is $3N$ dimensional. If the wavefunction provides the description of physical system, this fact would make it a wave in configuration space, rather than in three-dimensional space, and this was considered unacceptable: a field like that cannot represent a physical field. Here is what Lorentz writes to Schrödinger: "If I had to choose now between your wave mechanics and the matrix mechanics, I would give the preference to the former, because of its greater intuitive clarity, so long as one only has to deal with the three coordinates x, y, z. If, however, there are more degrees of freedom, then I cannot interpret the waves and vibrations physically, and I must therefore decide in favor of matrix mechanics."[17] Here are similar concerns expressed by Einstein in a letter to Lorentz dated May 1st, 1926: "Schrödinger's conception of the quantum rules makes a great impression on me; it seems to me to be a bit of reality, however unclear the sense of waves in n-dimensional q-space remains." Similarly, here is an excerpt from a June 18th, 1926 letter that Einstein sent to Paul Ehrenfest: "Schrödinger's works are wonderful – but even so one nevertheless hardly comes closer to a

[13] See Bacciagaluppi and Valentini (2009) for a very interesting take on these years.
[14] Bohr (1928).
[15] de Broglie (1923).
[16] See Allori *et al.* (2011) for an argument for this thesis. See also section 2.3.1 for more on this.
[17] Lorentz in Przibram (1967).

real understanding. The field in a many-dimensional coordinate space does not smell like something real."[18]

These quotes suggest that the problem of compatibility between quantum mechanics and scientific realism, what I have called the realism problem, *has already been identified* in 1926 as the *configuration space problem* almost ten years before the measurement problem: the wavefunction is not suitable to represent physical entities because it is a wave in configuration space, *regardless of whether there are microscopic or macroscopic superpositions*. Thus, the realism problem is not that there are unobserved macroscopic superpositions (the problem of adequacy), and it is not that we need a precise rule to get rid of them (the problem of precision). The realism problem is a *problem of completeness*: *the wavefunction is not suitable to describe physical objects, so we need something else to do this in its place*. Here is Schrödinger, almost ten years before his 1935 cat paper was published: "The direct interpretation of this wave function of six variables in three-dimensional space meets, at any rate initially, with difficulties of an abstract nature."[19] Also: "Of course this use of the q-space is to be seen only as a mathematical tool, as it is often applied also in the old mechanics; ultimately [...] the process to be described is one in space and time."[20] Interestingly, this argument can be tracked down historically also to Heisenberg, who has been reported to have said, very vividly, referring to Schrödinger's work: "Nonsense, […] space is blue and birds fly through it."[21] This expresses his unacceptability of a theory with no fundamental three-dimensional fields and with no fundamental three-dimensional physical space.

Heisenberg took the configuration space problem to be a reinforcement of his instrumentalism, and Lorentz was tempted to do the same, as the previous quote suggests. Instead Einstein looked for something which could represent physical entities, over and above the wavefunction. In 1927 he proposed a deterministic so-called 'hidden variable' quantum theory, in which he supplemented quantum theory with some other variable in addition to the wavefunction to represent matter in the theory.[22] He later retracted his paper before publications due to what he regarded as insurmountable difficulties, arguably having to do with the fact that for a system with two independent components his theory would not provide independent velocities.[23] In addition, in the discussion section of the 1927 Solvay conference, Einstein provided another argument against the completeness of quantum mechanics, dubbed in the literature Einstein's boxes.[24] He wants us to consider a particle traveling towards a narrow slit, behind which there is a curved detection screen. After the slit, because of diffraction, the wavefunction of the particle would propagate as a spherical wave, and thus hit the screen everywhere. Instead,

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[18] Both these quotes are taken from Howard (1990).

[19] Schrödinger (1926).

[20] Bacciagaluppi and Valentini (2009), 447.

[21] Bloch (1976).

[22] See Howard (1990), and Cushing (1994).

[23] See Belousek (1996), and Holland (2005).

[24] See Norsen (2017).

experimentally, we observe only one spot on the screen. Because of this, Einstein concludes, the description provided by the wavefunction cannot be complete. This thought experiment is very similar to one of the examples in the 1935 paper by Schrödinger before he mentions the cat example. He in fact starts with presenting the case of a radioactive source which could emit particles, one at a time, in all directions. If the description provided by the wavefunction were complete, and it would evolve according to the Schrödinger equation, then the particle would be detected in all the directions at the same time, which is not what happens. In my opinion this shows that *this completeness problem expresses the same worry expressed in the measurement problem thought as the problem of macroscopic superpositions*. This reading is also reinforced by the fact that in the weeks that preceded the publication of the cat paper, in a letter to Schrödinger, Einstein discusses an example which is strikingly similar to the cat thought experiment: "The system is a substance in chemically unstable equilibrium, perhaps a charge of gunpowder that, by means of intrinsic forces, can spontaneously combust, and where the average life span of the whole setup is a year. In principle this can quite easily be represented quantum-mechanically. In the beginning the ψ-function characterizes a reasonably well defined macroscopic state. But, according to your equation, after the course of a year this is no longer the case at all. Rather, the ψ-function then describes a sort of blend of not-yet and of already-exploded systems."[25]

Moreover, it was presumably in order to reinforce the conviction that the wavefunction is not suitable to be interpreted physically that Einstein proposed incompleteness proofs such as the one in the famous 1935 EPR paper.[26] Einstein, Boris Podolsky and Nathan Rosen (EPR) argue that if the Schrödinger evolving wavefunction is complete, then the measurement on one component of a pair of entangled particles travelling in opposite directions will instantaneously determine the state of the other component of the pair, regardless of their mutual distance. This implies superluminal influence, which is forbidden by relativity theory, from which EPR concluded that quantum mechanics is incomplete. Notice that there is a sense in which the problem of macroscopic superpositions (exemplified by the Einstein boxes in 1927, or the cat problem in 1935) and the EPR problem (also in 1935) are *the same problem* for Einstein.[27]

In any case, we know now that the EPR argument is flawed, in that it assumes that the world is local.[28] However, I contend, Einstein's attempt was to provide an argument for the incompleteness of quantum mechanics, in addition to the ones already provided. In 1935 Schrödinger reiterated these arguments for the same conclusion. If so, Schrödinger and Einstein arguably took *the problem discussed in the 1935 Schrödinger paper to actually illustrate the completeness problem, which is the problem to solve, as the configuration space problem exemplifies.* The

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[25] In Fine (1996).

[26] Einstein, Podolsky, and Rosen (1935).

[27] Part of the thesis that I am defending here, namely that for Einstein the problem of quantum mechanics is the completeness problem, is not very controversial, I think, and it has been defended by many, see e.g. Howard (1990), Belousek (1996), Fine (1996), Norsen (2017). However, I wish here to emphasize the connection with the configuration space problem, whose significance I think has been so far overlooked.

[28] See, e.g. Norsen (2017) and references therein.

examples by Schrödinger were taken to signify that the wavefunction represents our state of knowledge of the system, rather than the system itself, not because there are no macroscopic superpositions, but because there cannot be physical waves in configuration space. Schrödinger in fact writes: "I am long past the stage where I thought that one can consider the w-function as somehow a direct description of reality."[29]

Thus, if my reconstruction is correct, the measurement problem as the problem of completeness was known almost 10 year earlier motivated by the configuration space problem: if the wavefunction cannot be taken as a physical field in configuration space, then quantum mechanics has to be about something else. Thus for Einstein, *even if one solves the problem of macroscopic superpositions (that is, what people usually call the measurement problem and I have called the precision problem), one would not necessarily solve the configuration space problem* (if one solves it without not completing the description provided by the wavefunction).

I think this also shows that the pilot-wave theory can solve the problem of completeness only if one thinks of the *particles* not as addition to the wavefunction, but as a *necessary specification* of the nature of matter which the wavefunction could not provide. This is probably not in line of what David Bohm himself thought about his own theory;[30] however, this is what Einstein conceived a completion of quantum theory to be. Let me elaborate on this in the next subsection.

2.3.1. Objections to This Reading

Before we continue the discussion, let me address a potential difficulty with the reading that for Schrödinger and Einstein the realism problem is the completeness problem. If they thought the problem of quantum theory were a problem of completeness, then why did Schrödinger and Einstein dismiss this theory? Schrödinger knew about de Broglie's theory but instead of following this line of research, he continued with his wave mechanics and got rid of the particle component. Moreover Einstein, after an initial moment of enthusiasm, also dismissed it, even when it was proposed again by Bohm in 1952. In a May 3rd, 1953 letter to Mauritius Renninger Einstein wrote regarding Bohm's theory: "I do not believe that such a theory is tenable."[31] Einstein moreover actively argued against Bohm's theory, as he wrote a paper against it few years later. However, the argument was based on a misunderstanding of the theory, as later pointed out by Bohm.[32] These facts seem to be in direct conflict with my claim that for Einstein the realism problem was the problem of completeness and with Einstein's own attempt of finding a theory which would complete quantum theory.

Arguably Einstein's attitude against the pilot-wave theory was motivated by at least two reasons. First, the original proposal by Bohm assumed that the wavefunction was a physical

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[29] Schrödinger (1935): Schrödinger to Einstein, 19 August 1935; translated in A. Fine (1996), 82.
[30] See the reconstruction of Bohm's position in Brown and Wallace (2005).
[31] Quoted in Jammer (1982).
[32] See Belousek (1996).

fields, which Einstein rejected because of the unacceptability of thinking of a field in configuration space to be a physical field. Moreover, even if he does not explicitly mention this reason, the quantum potential in the pilot-wave theory is explicitly nonlocal, and thus Einstein had reasons to dislike it, as it directly conflicts with relativity. Daniel Belousek[33] has argued that by the time Bohm's theory was proposed Einstein had changed his mind regarding the possibility of completing quantum theory in a way which was compatible with relativity. He instead convinced himself that to make progress one would need a new theoretical framework, and that no completion of quantum mechanics could be successful. He expressed this in a November 14th, 1954 letter to Kuppermann: "I think it is not possible to get rid of the statistical character of the present quantum theory by merely adding to the latter, without changing the fundamental concepts about the whole structure." [34] Again: "I am inclined to believe that the description afforded by quantum mechanics is to be viewed [...] as an incomplete and indirect description of reality, that will again be replaced later by a complete and direct description. In any case one should be on guard, in my opinion, against committing oneself dogmatically to the schema of current theory in the search for a unified basis for the whole of physics."[35] So, presumably "he did not like Bohm's theory because he had merely supplemented rather than supplanted the received conceptual basis."[36]

As for Schrödinger, as anticipated, one could argue that he wanted to provide a theory without a double ontology of particles and waves, like de Broglie initially proposed, and this is why he developed a pure wave mechanics. He writes the following to Lorentz: "You mention the difficulty of projecting the waves in q-space, when there are more than three coordinates, into ordinary three dimensional space and of interpreting them physically there. I have been very sensitive to this difficulty for a long time but believe that I have now overcome it. I believe […] that the physical meaning belongs not to the quantity itself but rather to a *quadratic* function of it. […] If we now have to deal with N particles, then $\bar{\psi}\psi$ (just as ψ itself) is a function of $3N$ variables or, as I want to say, of N three dimensional spaces, $R_1, R_2, \dots, R_N$. Now first let R_1 be identified with the real space and integrate $\bar{\psi}\psi$ over $R_2, \dots, R_N$; second, identify R_2 with the real space and integrate over $R_1, R_3, \dots, R_N$; and so on. The N individual results are to be added after they have been multiplied by certain constants which characterize the particles (their charges, according to the former theory). I consider the result to be the electric charge density in real space."[37] That is, Schrödinger responded to Lorenz's configuration space problem by completing quantum theory. To the description of the wavefunction he added the specification that matter is described in terms of a charged density three-dimensional field given by the square root of the wavefunction. However, he soon realized that this description was empirically inadequate because particles, identified by such charged wave-packets in three-

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[33] Belousek (1996).
[34] Quoted in Fine (1986), note 67.
[35] Quoted from Howard (1985).
[36] Belousek (1996).
[37] In Przibram (1967): 55-56.

dimensional space, would become *very quickly delocalized*, because wave-packets spread. Moreover, he arguably found unacceptable that his theory still has the problem of macroscopic superpositions: whether in configuration space or in regular three-dimensional space, waves may combine into unobserved macroscopic superposition. Thus, one could maintain that he provided a variety of a many-world theory in which the wavefunction is replaced by a charged density wavefunction.[38]

2.3.2. The Primitive Ontology Approach and the Problem of Completeness

Regardless, people started to focus on the problem of macroscopic superpositions, so that the configuration space problem was not discussed anymore for a very long time, and realists did not longer question the idea of the wavefunction representing a physical field. However, in the last two decades, some authors have started resisting to the characterization of quantum theories in terms of a wavefunction ontology.[39] They propose that in all quantum theories material entities are not represented by the wavefunction but by some other mathematical entity in three-dimensional space (or four-dimensional space-time) which they dub the *primitive ontology* (PO) of the theory. Because of this, I think they can be considered as followers of Schrödinger and Einstein, as they think that the problem of quantum mechanics is the problem of completeness: one always needs an ontology represented by a mathematical object in three-dimensional space because, given that the wavefunction is a field in configuration space, it cannot represent the ontology of quantum theory. Therefore all quantum theories, if about the wavefunction, are incomplete, regardless of how the wavefunction evolves. Thus, not only the pilot-wave theory should be thought fundamentally as a theory of particles, but also the many-worlds theory and the spontaneous localization theory. They also are 'hidden variable' theories, in the sense that matter needs to be described by something else (in three-dimensional space) and not by the wavefunction. [40]

It should be clear by now that if one grants that quantum theory is complete, then one can complete however she wants. The pilot-wave theory is arguably the simplest way of doing this: the simplest type of ontology (particles), and the simplest evolution equation for the wavefunction. But it is not the only way to go. In fact, twenty years after the proposal of his spontaneous localization theory, Ghirardi introduced the idea that the microscopic description of matter is given by a (three-dimensional) matter density field defined in terms of the wavefunction, just like Schrödinger did, as mentioned in the last section. With two differences, however: first, Schrödinger identified the field with a charge density while Ghirardi with a mass density; and second the wavefunction in this theory evolves stochastically, as the

[38] This has been maintained by Allori *et al*. (2011).

[39] Dürr, Goldstein, and Zanghí (1992); Dürr, Goldstein, and Zanghì (1997); Allori *et al.* (2008); Allori (2013a); Allori (2013b). For a review, see Allori (2015); Allori (2019).

[40] Notice however, that they are hidden variable theories only in the sense that one cannot read their ontology in the formalism of quantum mechanics, which is, in this approach, fundamentally incomplete.

spontaneous localization theory prescribes.[41] This theory has been dubbed GRWm. However, there are other possibilities: Bell proposed an ontology of spatiotemporal events, dubbed the 'flashes,' so that the corresponding theory has been named GRWf.[42] Moreover, one can also consider a spontaneous localization theory of particles.[43] A similar attitude is adopted for the many-worlds theory, which are then divided into theories with a particle ontology, a matter density ontology, a flash ontology, and so on.[44]

In this approach the wavefunction is not an expression of our ignorance like in some of the approaches we discuss later (see the IT approach, section 3.2). Rather, it is best seen as law-like: in fact the role of the wavefunction, just like the potential or the Hamiltonian in classical mechanics, is to help implementing the law governing the spatio-temporal trajectories of matter PO.[45] This view fits particularly well with a Humean account of laws, according to which laws are axioms and theorems of our 'best system' of the world. Since the wavefunction is part of the axioms, it can be naturally regarded as a Humean law.[46]

3. …For Different Types of Realism

Scientific realists traditionally are taken to believe that physical theories which are not amenable to a realist interpretation are unsatisfactory. This is what led them, in the quantum domain, to look at the measurement problem as the realism problem. However, as we have just seen, one can identify different types of worries connected with the same problem. In this section I show that different types of scientific realists will tend to think of the problem in the measurement problem differently.

I distinguish between two broad kinds of realist attitudes: a robust kind, and a more relaxed kind. In fact, in general one defines a scientific realist as someone who believes that scientific theories can be taken as approximately true descriptions of the world. However, one can disagree about what a theory needs to do in order to provide such an accurate description of the world. Some, which I call *relaxed* realists, may think that it is sufficient to account for the macroscopic regularities. Others, which I call *robust* realists, instead may insist on something more, namely a fundamental, dynamical, explanation of why such regularities actually happen.

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[41] Benatti, Ghirardi and Grassi (1995).

[42] This theory was first proposed by Bell (1987b), and then adopted in Tumulka (2006), who developed a relativistic extension.

[43] Allori (2020a).

[44] Allori (2019), and references therein.

[45] Dürr, Goldstein, and Zanghí (1997); Goldstein and Teufel (2000); Goldstein and Zanghí (2013); Allori (2018).

[46] See Miller (2014); Esfeld (2014); Callender (2015); Bhogal and Perry (2017). There are other ways in which someone could think of the wavefunction, broadly speaking, as nomological. One can think of the wavefunction as a property which expresses some non-material aspect of the particles, see Monton (2013). Similarly, one can endorse a dispositional account where laws are understood in terms of dispositions, which in turn are described by the wavefunction, see Esfeld *et al.* (2014).; M. Suàrez (2015). For objections and replies, see Allori (2019) and references therein.

3.1. The Completeness Problem and Robust Realism

I have argued above that Einstein was convinced that quantum mechanics was incomplete since the 1920s, and it was because of this reason that he thought it is incompatible with realism. To summarize the result of the last section, he provided the following reasons, in temporal order starting from 1926 until 1935, for the conclusion that quantum mechanics is incomplete: 1) if it were complete then the wavefunction would be a 'wave' in configuration space, which cannot be interpreted as something vibrating, and that is unacceptable; 2) if it were complete then the world would be nonlocal, and that is unacceptable; 3) if it were complete then we would have macroscopic superpositions, and that is unacceptable. We know that the second argument is not sound, because Bell's inequality, derived from the EPR argument, is experimentally violated. However, while the latter of the other two arguments focuses on what we observe macroscopically, the first one focuses on determining which objects can be suitable ontologies for a theory. Einstein is not explicit about this but it seems clear to me that he required a satisfactory ontology to be in three-dimensional space, given that he considered a wave in configuration space unphysical.

This view is robustly realist: Einstein was looking for something to account for a microscopic dynamics to explain why the quantum rules work. This is supported also by the fact that Einstein was a proponent of the statistical interpretation, as he writes in a letter after the publication of Schrödinger's paper: "this example shows exactly that it is reasonable to let the ψ -function correspond to a statistical ensemble that contains both systems with live cats and those with dead cats."[47] That is, quantum theory works but we do not know why. This means that we are missing something in the microscopic description.

Both Einstein and Schrödinger wanted to make quantum mechanics compatible with scientific realism by providing a fundamental, microscopic, description of the quantum world, rather than merely suppress the macroscopic superpositions. They thought that the problem of realism was the configuration space problem which in turn pointed to the fact that something else must describe physical reality, if not the wavefunction. This amounts not merely to solve the measurement problem (that is, the problem of precisely eliminating macroscopic superpositions) but to solve it *in a very specific way,* namely by completing quantum theory. Thus, the spontaneous localization theory and the many-worlds theory would not be an option for them. Instead, as we will see in the next section, a relaxed realist would not necessarily look for any additional information to complete the description of the wavefunction. In fact, they would allow for instance the elimination of macroscopic superpositions by suitably modifying the Schrödinger equation.

3.2. The Adequacy Problem and Radical Relaxed Realism

In the literature on quantum foundations, traditionally the scientific realist has been opposed to the instrumentalist: while the latter cares only about reproducing experimental outcomes, the

[47] Quoted from Fine (1996): 84.

former cares about understanding the nature of reality which gives rise to such outcomes. And traditionally the realist has advocated that the realism problem is the measurement problem. In other words, the problem which needs a solution in order to have information about the nature of reality from quantum theory is the problem of precisely eliminating macroscopic superpositions. However, there are those who call themselves realists without seeing any need of solving the precision problem. That is, they think the problem is merely the one of accuracy, and thus they find nothing wrong with the solution provided by von Neumann. They take the orthodox view at face value, and then trying however to make realist sense of it. These can be thought of a radical variety of the class I have dubbed relaxed realist.

Bub and Pitowsky, who propose the so-called information-theoretic (IT) interpretation of quantum theory,[48] can be seen as an example of this type of realism. The basic idea is that quantum theory is to be understood as laying out a set of constraints imposed on the empirical data. They are realist about the fact that these data exist objectively and mind-independently, but they do not think they need to tell any additional story about how these data are generated. Building on this, James Ladyman and Don Ross[49] defend ontic structuralism, according to which macroscopic entities are not composed of microscopic constituents, and what they call rainforest realism. In their view objects, both at the microscopic and the macroscopic level, are seen as real patterns, defined by their usefulness.[50] And this is true also for what we call 'particles:' they are not fundamental objects but just useful fictions. Also, measurement devices and measurement results are not analyzed in terms of more fundamental entities (there aren't any) but they are understood as effective descriptions. Finally, one can also think of QBism, also known as Quantum Bayesianism,[51]as a radical relaxed realist approach: "what is the stuff of the world? QBism is so far mostly silent on this issue, but not because there is no stuff of the world. The character of the stuff is simply not yet understood well enough. Answering this question is the goal, rather than the premise."[52] They think of quantum theory as providing constraints on measurement outcomes, just like the IT approach and rainforest realists Be that as it may, for them the realism problem is a *problem of empirical adequacy*: we do not see superimposing experimental results. This can be solved by the collapse postulate, and it does not matter whether it is a precise rule or not. This is because in IT and QBism measurements are unanalyzable primitives, while in rainforest realism nothing is primitive. Once such a rule to 'kill' macroscopic superpositions is in place, the realism problem (given that it is a problem of adequacy) not only is solved, but it is 'dissolved.'

Usually, these attempts are taken as unsuccessful by the robust realist who claims these kinds of radical relaxed realism are not realist enough. Indeed, as described, the radical relaxed realist

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[48] Bub and Pitowsky (2010)*t*.
[49] Ladyman and Ross (2007); Ladyman, and Ross (2013).
[50] Ladyman (2016). This view follows the ideas of Dennet (1991) and Wallace (2003).
[51] Fuchs (2010); Fuchs, Mermin, and Schack (2014); Fuchs (2017) and references therein.
[52] DeBrota, and Stacey (2019).

just described is a realist which is borderline instrumentalist.[53] However, the instrumentalist will presumably say that physical theories are not the kind of things which can give us a description of what is unobservable, while the radical relaxed realist will not deny that. Indeed, she will acknowledge that there are many possible microscopic descriptions for the same macroscopic phenomena. But, as Ladyman and Ross would say, because of underdetermination concerns, one should remain agnostic about which the correct one is. Or, as Bub and Pitowsky would maintain, no microscopic description is needed to make the explanation satisfactory (see section 5). Regardless, this agnosticism makes them realists, in the sense that they do not deny that a microscopic reality exists, but they are not robust realists, as they do not provide any microscopic description.

3.3. The Precision Problem and Non-Radical Relaxed Realism

In any case, as anticipated, a robust realist will still object that the view of the radical relaxed realist is fundamentally ambiguous, as it is unclear when one applies the quantum rules: what is a measurement process? Relatedly, the previous accounts work under the assumption that measurement processes are unanalyzable, and that the Schrödinger dynamics is not universal. This is connected to the concern that it is vague and imprecise what a macroscopic phenomenon is. Thus, someone may ask form more, without however necessarily going robust. That is, someone could still *care about the adequately reproducing the phenomena but she could also care about having all physical objects and processes being governed by a unified, precise dynamics.* In other words, one would like to have an equation of motion for both the fundamental entities and the measurement processes. So, while for the radical relaxed realists we saw in the previous section the realism problem as the adequacy problem dissolves when adopting the collapse rule, for this other type of non-radical relaxed realists there is still the *problem of precision*. This is a realist account, but still relaxed because it deals with the phenomena at the macroscopic level. In other words, this is a variety of relaxed realism in which one cares also of the adequacy of the experimental predictions (similarly to the radical relaxed realists), but one solves the measurement problem to make the quantum theory precise as the pilot-wave theory, the spontaneous localization theory, and the many-worlds theory do (contrary to the radical version of relaxed realism). This type of non-radical relaxed realism provides a middle ground between the robust realism (which looks for something to complete quantum theory) and the radical relaxed realism (which is content with the collapse rule).

Let me notice, however, that it is a very peculiar position to hold: how did a realist end up focusing on the phenomena (macroscopic superpositions) and not on the microscopic dynamics? Instrumentalists care about these things. Should not the realist care for more? Indeed, this is the position of the robust realism who cares about more than just eliminating macroscopic superpositions: she looks for the microscopic dynamics which generates what we

[53] SeeEgg (2019) for a set of arguments that some implementations of this type of realism, such as the one of Ladyman and Ross and of Bub and Pitowsky (discussed below), do not deserve to be labelled realism.

observe at the macroscopic level. She uses the measurement problem to make the point that quantum mechanics is fundamentally complete, rather than to claim that it is enough to eliminate the macroscopic superpositions (because for them it is not). Now the question is: why think that the measurement problem is the realism problem? Why care about macroscopic superposition at all? In other words, it seems to me that the measurement problem is fundamentally a problem for the instrumentalist, rather than for the realist. This is somewhat paradoxical, because it is the realist who brings up the measurement problem as a problem for quantum theory. That the measurement problem is an instrumentalist problem is clear by noticing its name: the notion of measurement is involved in the formulation of the measurement problem, and the problem is the macroscopic superpositions, so that it is not a problem at the microscopic level. It is a problem of *which* measurement outcomes we could obtain, not a problem about *why* we obtain them. We do not worry about what kind of microscopic description could give rise to such macroscopic measurement outcomes. As such, this attitude is positivistic in spirit: all we care is about what we can observe. However, one could think that this instrumentalist influence disappears when the problem is presented in the form of the cat experiment as proposed by Schrödinger. When the story is discussed in these terms the notion of measurement never appears. So, has the positivistic influence gone away? I think not. In fact, while Schrodinger was not an instrumentalist, in the subsequent presentations of the cat problem there is a sense in which some positivistic flavor is still there. In fact even in the formulation of the cat problem it is not suggested we should have a problem with microscopic superpositions.[54] The reason for finding microscopic superposition unproblematic is that *we do not experience microscopic entities*. Here is where some form of instrumentalism sneaks back in: we give up on trying to make sense of microscopic superpositions, we are merely worried of 'killing' the superpositions at the macroscopic level. For instance, this is what David Wallace writes: 'This […] implies a very weird view of the microworld, but since that world is not directly observable such weirdness is not […] a problem."[55]

The fact that this problem has some instrumentalist flavor should not be surprising, given the historical context. Instrumentalism and positivism as philosophical movements were popular at the time, and as it is well known the founding fathers of quantum theories were heavily influenced by them. As mentioned, Heisenberg developed his matrix mechanics, a purely computational machinery to derive experimental outcomes, and he got convinced that the microscopic world was indescribable. Similarly, Bohr developed his doctrine of wave-particle duality and complementarity, to argue for the possibility of a complete understanding of physical reality. He writes: "The quantum theory is characterized by the acknowledgment of a fundamental limitation in the classical physical ideas when applied to atomic phenomena."[56] Moreover, also more realist inclined philosophers such as Einstein had to convince the rest of

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[54] One may think this is not a problem because we can think of nuclei as waves, and waves superimpose. However, that is not the case: we cannot think of them as waves because in experiments they show both features of waves (interference) and of particles (precise location).

[55] Wallace (2003).

[56] Bohr (1928).

the community in their own, positivistic terms, so that the measurement problem could be seen as summarized along the lines of the following silly dialogue: “Do you care only about what you observe?” “Yes” “Do you observer macroscopic superpositions?” ”No…” ”Here you go.” Indeed, even Einstein was under positivistic influences: just think of the role of observability in eliminating the aether or defining the notion of simultaneity in the theory of relativity. In any case, a more thorough historical investigation would be needed, but the situation is that while Einstein went robust in his realism, other realist inclined philosophers focused on the problem of macroscopic superpositions and left the configuration space problem untouched, until recently.

3.4. Which Problem?

So, to sum up, quantum theory is not a realist theory, but it is rather a recipe for predicting results, without explaining where they are coming from. People think that it is possible to turn this theory into a realist one by solving some problem, but I argued that they disagree about which problem needs solving. On one end of the spectrum we have robust realists such as Schrödinger and Einstein took the realism problem of quantum theory to be the problem of completeness. On the other end we have radical relaxed realists instead think that nothing more needs to be done once the collapse is postulated to happen: in this way, the traditional solutions of what everyone takes to be the measurement problem (the pilot-wave theory, the spontaneous localization theory, the many-worlds theory) are not needed. In the middle we have non-radical relaxed realists who think one needs to solve the realism problem such as to make the dynamics precise along the lines of the pilot-wave theory, the spontaneous collapse theory and the many-worlds theory. Therefore among those who think that the realism problem needs a precise solution, the robust realist has less options to solve the realism problem than the relaxed realist: in fact there are solutions of the precision problems which are not solutions of the incompleteness problem. These solutions, such as the many-worlds theory and the spontaneous localization theory, will be available to the relaxed realist but not the robust one. This is compatible with the received view among philosophers and physicists interested in the subject which *agree* that a quantum theory which aims at providing a realist description of the world needs to solve the precision problem. However, since the precision problem allows more solutions than the completeness problem, they generally *disagree* with Schrödinger and Einstein’s assessment that quantum theory is necessarily incomplete. Thus, they point out that supplementing the description provided by the wavefunction is only one of the possible ways in which one could solve the realism problem, as one could alternatively modify the Schrödinger evolution, or embrace macroscopic superpositions. The following table shows how different types of realism map into different types of realism problem, and thus into different acceptable solutions.

Table 1:

Type of Realism	Type of Problem	Acceptable Theory

Robust	Completeness	Hidden variable type
Non-radical Relaxed	Precision	Hidden variables, modified Schrödinger dynamics, many-worlds
Radical Relaxed	Adequacy	von Neumann rule

4. The Big and the Small in the Measurement Problem

In the last two sections, I have distinguished two different types of realism, relaxed and robust, to show how different people focus on different problems when trying to solve the realism problem of quantum mechanics. Let's set aside the radical relaxed realists for a moment. While (non-radical) relaxed realists look at the realism problem as the problem of precision, robust realists look at the completeness problem. I finally have concluded that, because of this, the (non-radical) relaxed realist has more options than the robust realist to make quantum theory compatible with (their respective form of) realism.

Interestingly, Pitowsky made a related distinction.[57] He distinguished between a *small and a big measurement problem*. The big problem is what I have called the completeness problem, while the small problem is what everyone typically calls the measurement problem, namely the precision problem. In fact, the former is the problem of providing a dynamical explanation of why particular experiments have the outcomes they do; while the latter is the problem of explaining how 'classical' macroscopic measurement outcomes arises from a quantum measurement process, namely a problem about empirical adequacy. Bub and Pitowsky claim that it is usually thought that realist have to solve the big problem.[58] However, they argue that this is too much to ask, as someone can be a realist only by solving the small problem instead. This is compatible with my reading, *provided that they do not endorse the same type of realism*: the robust realist requires a microscopic dynamics, and thus she looks for completing quantum theory, while the relaxed realist only sees the need of solving the small problem. Notice again that someone who solves the small problem is a relaxed realist, but *still a realist*: she is being agnostic about what the microscopic description is, and is content with the possibility of accounting for the description of the phenomena (see section 5).

In this regard it is worthwhile recalling in the next section Bub and Pitowsky's argument for the conclusion that many-worlds theorists are relaxed realists. Notice that Bub and Pitowsky do not say this, as they merely call them realist, and they use this result to argue that their IT approach is realist too. Instead I wish to underline that the type of realists many-worlds theorists end up being is much weaker than usually advertised, and this has consequences. While some prominent supporters of the many-worlds theory have endorsed a view called spacetime state realism, a view which takes the states associated to spacetime regions as

[57] Pitowsky (2007).
[58] Bub and Pitowsky (2010).

fundamental,[59]some others have maintained that the theory is about the wavefunction. This view is commonly called *wavefunction realism* and it is defended most notably by David Albert and Alyssa Ney. [60] As we will see later, wavefunction realists take themselves to be robust realists, as they think quantum theory gives us a picture of reality through the wavefunction. They also claim that, while the framework of the pilot-wave theory does not fit well with their approach, the many-worlds theory and the spontaneous localization theory are instead a better fit.[61] If so, and if Bub and Pitowsky are correct, then wavefunction realists endorsing the many-worlds theory are relaxed realists, contrary to their proponents' claims of being robust realists. In section 4.2 I also show how the same type of argument can be constructed in the spontaneous localization theory framework.

4.1. The Many-Worlds Theory as a Relaxed Realist Theory

Bub and Pitowsky argue that the many-worlds theory solves only the small problem (the empirical adequacy one) rather than the big problem (the completeness problem) but still it is considered by everyone a realist theory. If so, they continue, since their IT approach is capable of solving the small problem too, it should be considered realist as well.

They ask us to think about what the many-worlds theory is doing: they are accepting that there are macroscopic superpositions, but we do not see them because, due to the interaction with the environment (that is, because of the phenomenon of decoherence), the different terms of the superposition, never interact with one another and thus are describable 'as if' they live in different 'worlds.' More technically, the wavefunction decomposes into a linear superposition describing an emergent branching structure of non-interfering quasi-classical 'worlds,' identified by our experience, weighted by the Born probabilities. The alternative outcomes of a measurement process are associated with different branches in the decomposition of the wavefunction. Thus Bub and Pitowsky write: "The basic problem for the Everettian is to 'save the appearances,' given the radical difference between our experience of a stable macroworld and the ontological assumption. The dynamics of decoherence yields an emergent weighted branching structure of quasi-classical histories at the macrolevel." The many-worlds theory needs to explain how measurements have definite results, and its reply is that they do not. Because of this, the many-worlds theorist solves the small problem while she "too, regards the 'big' measurement problem as a pseudo-problem, because the Everettian rejects the assumption that measurements have definite outcomes, in the sense that one particular outcome, as opposed to other possible outcomes, actually occurs in a quantum measurement process."[62] Many-worlds theorists also have the so-called 'problem of probabilities.' In fact, the theory is deterministic but there is a sense in which someone on a branch will be uncertain about which branch she will subsequently occupy. She will have some degrees of belief (credences) about the

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[59] Wallace and Timpson (2009).
[60] Albert (1996); Albert (2013); Albert (2015); Ney (2012); Ney (2013); Ney (2015); Ney (2017); Ney (forthcoming).
[61] Ney (forthcoming).
[62] Bub and Pitowsky (2010).

measurement outcomes, even though they occur on different branches. To solve the measurement problem one needs to postulate that what someone believes (rationally) is identical with what is specified by the quantum probabilities, namely the weights of the different branches. Many-worlds theorists have given several arguments for why this would be the case using additional assumptions which can be justified as rationality constraints.[63] Bub and Pitowsky argue that these strategies are compatible with solving the small, rather than the big, problem: "what has to be explained is how uncertainty or caring makes sense when all alternatives occur relative to different branches, and how the quantum weights—which are a feature of the quantum state, i.e., the ontology—are associated with the credence function or caring measure of rational agents." Therefore, since the many-worlds theory solves the small measurement problem rather than the big one, it is best seen as a relaxed realist, rather than a robust realist theory.[64]

Also, let me notice that the conclusion that the many-worlds theorist is a relaxed realist who cares about the macroscopic appearances is supported by arguments developed in some other type of literature. Harvey Brown and David Wallace[65] for instance have argued that the pilot-wave theory is actually a many-worlds theory in denial: it has both waves and particles, but they argue that the theory can solve the measurement problem even without the help of the particles. The particles, that is, play no role. In fact, they argue, the measurement problem is solved if one can account for the observed measurement results, and this can be done with the wavefunction alone, as in the many-worlds theory: "The Everettian ontology is now robustly monistic — the wave-function constitutes the whole of reality — and it is the Bohm theory which is ontologically excessive." However, this makes sense only if one thinks of the realism problem as the problem of getting rid of macroscopic superpositions. As long as one thinks of the wavefunction as representing objects, macroscopic superpositions will be there for the pilot-wave theory too, regardless of their particles component, just like they are there in the many-worlds theory. The many-worlds theory solves the problem with the techniques summarized above, which are also needed by the pilot-wave theory. So, according to Brown and Wallace, the particles are doing nothing to solve the problem. This is because the problem that Brown and Wallace have in mind is the problem of precision, namely the problem that a relaxed realist would find interesting. However, if one thinks of the pilot-wave theory as solving the problem of completeness rather than solving the precision problem assuming that the wavefunction is not a physical field, then Brown and Wallace argument does not get off the ground: in that theory the wavefunction does not represent matter, the particles do, and this is because the problem is not that there are macroscopic superpositions but that the description provided by

[63] See e.g. the following: Deutsch (1999); Wallace (2003); Saunders (2003); Greaves (2004); Wallace (2006); Wallace (2007); Greaves (2007); Saunders and Wallace (2008).
[64] For criticism of Bub and Pitowsky, see Timpson (2010).
[65] Brown, and Wallace (2005).

the wavefunction is incomplete. In this way we can also explain why the pilot-wave theory is favored by the proponents of the so-called primitive ontology programme.

At any rate, Bub and Pitowsky argue, as we have just seen, that if the many-worlds theory solves the small problem but still is considered a realist theory, so are they. In our terminology, they have showed that the many-worlds theorists are relaxed realists. However, I would not say that they are *radical* relaxed realists, *contra* the IT approach. In fact, the proponents of the IT interpretation think of the wavefunction not in terms of providing a fundamental description of reality. Rather they conceive of it as *epistemic*: according to them the wavefunction is a reflection of our ignorance, and does not describe the world. In contrast, the information provided by the wavefunction realists endorsing the many-worlds theory think of the wavefunction as able to suitably describe physical systems.

I show in the following section that the same can be said for the spontaneous localization theory. The conclusion follows almost immediately from what I have been arguing so far: as it is a solution of the realism problem which does not complete quantum theory, it is only a relaxed rather than robust realist theory. In more detail I show how, by analyzing the historical roots of the theory, being such a relaxed realist theory is compatible with the intent of its proponents. Moreover, even if one thinks of the theory in more realist terms as a theory about the wavefunction, still it is not a robust realist theory.

4.2. The Spontaneous Localization Theory as a Relaxed Realist Theory

The spontaneous localization theory can be seen as a solution of the measurement problem along the lines of von Neumann's collapse rule. However, as we have seen, the collapse postulate is vague in nature, as it is not clear wat a measurement process is. Accordingly, some people tried to develop theories which would modify the Schrödinger dynamics without fundamentally invoking the notion of measurement or observer. Such a theory would not only be empirically adequate but also would solve the problem of precision. The spontaneous localization theory proposed by Ghirardi, Rimini and Weber provides a way of providing a unique evolution equation for the wavefunction as to make precise the von Neumann approach. In this theory the wavefunction does not evolve according to the Schrödinger equation but stochastically localizes in random places at random times so that macroscopic objects are never found in superpositions.

4.2.1. The Spontaneous Localization Theory as a Solution of the Problem of Precision

The idea is that systems are subject at random times to random and spontaneous localization processes around appropriate positions. When the localization happens in, say, x_i then the wavefunction is instantaneously multiplied by a Gaussian function (appropriately normalized) with width d, which makes the wavefunction 'lump' around x_i. It is assumed that localizations occur at randomly distributed times, according to a Poisson distribution with frequency f. The theory is also constructed so that the spontaneous localization mechanism is enhanced by increasing the number of degrees of freedom of the system, which amounts to the suppression of macroscopic superpositions. The theory has two new constant of natures: the width d of the Gaussian distribution which acts during localization, and the frequency f of localization. They have been chosen, via empirical considerations, as follows: $f = 10^{-16}s^{-1}$, and $d = 10^{-5}cm$. That means that a microscopic system undergoes a localization, on average, every hundred million years, while a macroscopic one undergoes a localization every 10^{-7} seconds. This theory does not have the problem of vagueness and imprecision of von Neumann, but shares the same spirit: nothing is added to the specification provided by the wavefunction and as such the theory is still driven by recovering the macroscopic appearances rather than providing a microscopic fundamental, description.

To support the idea that this theory comes from the desire of giving to the quantum rules a precise method of application rather than figuring out a microscopic ontology for the quantum phenomena, let's briefly review the origin of spontaneous localization theory. Philip Pearle, Nicolas Gisin, Lajos Diosi, and others developed nonlinear and stochastic modifications of the Schrödinger equation without however being able to provide a general account. [66] The breakthrough happened when Ghirardi, Rimini and Weber proposed their theory as described above, in which a nonlinear and stochastic dynamic is supplemented by some suitable parameters. By their own admission,[67] the goal of all these modifications was to solve what they called the *macro-objectification problem*, namely to 'kill' the macroscopic superpositions *without affecting the microscopic ones*, and the efforts of all these scholars were focused on finding a mechanics that would make this happen.[68] This is what I have called the problem of precision. It was noted that nonlinear but deterministic modifications of the Schrödinger equation would give rise to faster-than-light signaling which would then make the theory fundamentally incompatible with relativity.[69] Moreover, they had the problem of the preferred basis: which, among the different representation of the wavefunction, is 'the correct' one? This was solved by taking the position basis, with the justification that "the most embarrassing superpositions, at the macroscopic level, are those involving different spatial locations of macroscopic objects."[70] In this way the collapse postulate was 'built' in the evolution equation as an approximate position measurement. Moreover, the original 1986 proposal was in terms of density matrices and was constructed by improving on a model by Alberto Barchielli, Lodovico Lanz and Giovanni Prosperi,[71] who developed a formalism to describe macroscopic objects. In this sense, the title of the paper, namely "Unified Dynamics for Microscopic and Macroscopic Systems" should not be seen as indicating that they propose a microscopic theory which would explain the experimental outcomes; rather, it is best seen as conveying the desire of making a precise

[66] Pearle (1976); Pearle (1979); Gisin (1984); Diosi (1986).
[67] See e.g. Ghirardi (2018).
[68] This is Pearle's trigger problem (Pearle 1989).
[69]Gisin (1989).
[70] Ghirardi (2018).
[71] Barchielli, Lanz, and Prosperi (1982); Barchielli, Lanz, and Prosperi (1982).

quantum theory, intended as a tool for generating experimental outcomes, applicable at all scales, without the vagueness of figuring out what counts as a measurement process. The only difference between the von Neumann's collapse rule and the spontaneous localization theory is that the latter is precise while the former is not. *Neither one is providing a reason why experimental outcomes are what they are.*

4.2.2. The Spontaneous Localization Theory as a Theory about the Wavefunction: Bare GRW

In the reading that I just proposed people originally looked for appropriate modification of the Schrödinger equation as a way to make precise the quantum collapse rule, not necessarily getting us closer to a better understanding of the microscopic quantum ontology that would dynamically explain the success of such a recipe. Thus, given that quantum theory *a-la* von Neumann was at best a relaxed realist solution, so it is the spontaneous localization theory. At least, this is the way in which the theory was originally proposed.

However, regardless of how this theory came to be proposed, it has been interpreted by most scientific realists at face value, namely as a theory which is supposed to describe both microscopic and macroscopic objects, or in general the fundamental nature of reality. Since the wavefunction provides the complete description of any physical system, it seems natural to think of it as the mathematical object representing physical systems at all scales, wavefunction realists maintain. For the sake of clarity, in this paper I will call 'spontaneous localization theory' the theory originally proposed by Ghirardi, Rimini and Weber, while I will call 'bare GRW' the theory as interpreted by the wavefunction realist. [72]

In bare GRW the wavefunction is everything there is, and as such describes physical entities. One of the motivations for such a view is straightforward: as scientific realists we want to solve the realism problem understood as the precision problem, and the spontaneous localization theory provides one way of doing that. If now we want to read the ontology of the theory what we do is look at its fundamental equation. In classical mechanics the fundamental equation is Newton's equation, which is an equation for particles. Form this we infer that the theory is about the behavior of particles. Similarly, in this theory the fundamental equation is the modified Schrödinger equation, which is an equation for the evolution of the wavefunction. Because of this we are justified in inferring that this is a theory about the behavior of the wavefunction. Moreover, this theory seems to be the simplest solution of the measurement problem, as it does not add any ontology to quantum theory, as the pilot wave theory does, or a set of additional 'worlds,' as the many worlds theory does. Indeed, as we already mentioned, wavefunction realists argue that the spontaneous collapse theory provides the best framework, together with the many-worlds theory, to develop their view. In fact, their strategies applied to the pilot-wave scenario would suggest there are two fundamental spaces, one where the particles live, and one where the wavefunction lives, or even more complicated scenarios, and

[72] This terminology comes for Albert (1996) and has been taken up again in Allori *et al.* (2008).

there would be an additional problem of makings sense of the interaction between the two spaces.[73] In addition, it has been argued that wavefunction realism is the only picture in which the world is local and separable. The idea is that, since fundamentally all that exists is the wavefunction, there are no other facts that fail to be determined by local facts about the wavefunction. [74]

Be that as it may, given it is a theory about the behavior of the wavefunction, even if it solves the problem of precision, bare GRW suffers from the configuration space problem. Thus, it is not a solution of the completeness problem. However, the wavefunction realist needs to recover the macroscopic appearances from the description in configuration space provided by the wavefunction. Specifically, she needs to explain the appearance of three-dimensionality of space and of objects, the three-dimensional localization of these three-dimensional objects, and so on. In general, the solution strategies adopted by wavefunction realists are to argue that the three-dimensional world is *emergent or derivative*. In other words, the wavefunction realist needs to derive the appearances (three-dimensionality, localization, and regular macroscopic properties) in terms of the wavefunction and the quantum rules. In this framework there are two main attempts to recover the macroscopic objects from the wave-function.

The first is due to Albert[75] and based on a *functionalist reduction* of three-dimensional microscopic objects from the wavefunction, what he calls functional enactment.[76] The idea is that it is possible first to functionally define what it means to be a three-dimensional object. Then it is also possible to show that the wavefunction can play the role of a three-dimensional object. This functional reduction can give rise to microscopic three-dimensional objects, which then can be understood as usual, in particular as composing macroscopic objects. To see how that works, assume bare GRW is true, and that the wavefunction spontaneously collapses and bunches up around a given location in a three-dimensional subspace of the wavefunction's space. Albert introduces a bunch of functions, called 'shadows,' each of which assigns to each such location the corresponding wavefunction amplitude. He then argues that these shadows are capable of playing the functional role of three-dimensional microscopic objects, and then it is "the relatively stable coagulations of subsets of these shadows" that play the role of tables and chairs. [77] This is because to be a three-dimensional object is to behave as one. That is, to have one's behavior over time depends on changes in position and inter-particle distances in three dimensions.[78] The temporal behavior of an object is described by an operator called the Hamiltonian, and Albert thus argues that the wavefunction shadows are capable of playing this functional role given the right Hamiltonian.

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[73] See Ney (forthcoming).

[74] See Loewer (1996), Ney (forthcoming) and references therein.

[75] Albert (2015).

[76] Albert (2013, 2015).

[77] Albert (2015): 130

[78] Albert (1996, 2013, 2015).

Ney[79] is critical of this approach, as she points out that in this reading there is no common three-dimensional space for inter-particle interactions. She therefore proposes that symmetries can help pick out, among all the possibilities, the three-dimensional world as privileged: she observes that only a three-dimensional 'decomposition' of the wave-function (as opposed to any other kind of decomposition) can explain symmetry properties, and because of this reason it makes sense that we represent our world three-dimensionally rather than (say) two-dimensionally. Thus in her view three-dimensional objects exists, not as an additional postulate of the theory but as derivative from the wave-function when considering symmetry properties as fundamental facts about the world.

The interesting question at this point is: what kind of realism is endorsed by the supporter of bare GRW? Following Bub and Pitowsky, wavefunction realist solves the small problem, which is the problem of precisely eliminating macroscopic superpositions, and because of that they are relaxed realists. In fact they need to *recover appearances*, and they do so by providing constraints that the fundamental ontology obeys, not a dynamical explanation of why these constraints obtain. The fundamental ontology, the wavefunction, is not microscopic in any meaningful sense, and the objects of our experience which we call macroscopic are not composed of microscopic entities. Rather they are either three-dimensional shadows defined functionally or identified using symmetry considerations. If so, wavefunction realists who endorse theories such as the spontaneous localization theory (but also the many-worlds theory) as theories of the wavefunction are *relaxed realist in denial.* It is clear from the writings of the wavefunction realists that they think of themselves as being robust realists, at least in the sense that they care about the fundamental nature of reality: they call themselves wavefunction realists and they take the wavefunction 'ontologically seriously' as opposed to the epistemic inclinations of the proponents of the IT interpretation or of QBism. However, they end up endorsing a position which is too similar to the relaxed realist that they would arguably like: in fact they reproduce appearances and systematize the phenomena just as the relaxed realists do.

Let us see in the next section what the situation is in more details using the distinction between principle and constructive theories. Anticipating the conclusion, the main thesis is that wavefunction realists endorse a hybrid position between robust realists, who tend to like constructive theories, and relaxed realists, who instead favor principle theories. In fact the wavefunction realists are relaxed realists like Bub and Pitowsky but in contrast to them they care about the dynamics: since they want to solve the precision problem, they care about having a unified microscopic-macroscopic precise dynamics. On the other hand the explanation they provide is not similar to the one provided by the robust realist, as it is not dynamical in nature, since appearances are recovered in other ways, by constraints posed on the fundamental ontology.

[79] Ney (forthcoming).

5. Principle Theories, Constructive Theories and Scientific Realism

The wavefunction realist claims that the fundamental description of reality provided by bare GRW is in terms of the wavefunction, and that is the way of interpreting the theory realistically. However, as we have seen, this understanding is incompatible with what suggested by Einstein and Schrödinger, as in their opinion the wavefunction needs completion. In response the wavefunction realist will surely argue that it is obviously the case, and that it is why bare GRW is another, fundamentally different, way of solving the problem of precision. They will argue that *this* is the realism problem, not the problem of completeness. In the rest of the paper I argue that this attitude regarding which problem is most important reflects a distinctive way of understanding the nature and the explanatory structure of theories. I argue in general that relaxed realists, namely those who think the realism problem is the problem of precision, are likely to be principle theorists, while robust realists, who think the realism problem is about completeness, instead are likely constructive theorists.

5.1. The Principle/Constructive Distinction

To use a locution introduced by Einstein, one can distinguish between constructive and principle theories. I argue in this section that, while the pilot-wave theory can be easily seen as a constructive theory, the spontaneous localization theory is more similar to a principle theory, and bare GRW is a hybrid. More precisely, I argue that a robust realist like Einstein and Schrödinger, who thinks that quantum theory is not complete, will like constructive theories; while a relaxed realist, namely someone is open to the possibility that the theory needs no completion and it could be made precise and satisfactory by modifying the Schrödinger equation, will be more inclined to like principle theories.

Principle (or kinematic) theories are formulated in terms principles, which are used as constraints on physically possible processes, as in thermodynamics (e.g. 'no perpetual motion machines'). They are 'principle' theories because they spell out principles that the phenomena need to conform to, and they are ' kinematic' theories because the explanations they provide do not involve equations of motion but rather they do not depend on the interactions the system enter into. Instead, *constructive theories involve the dynamical reduction* of macroscopic objects in terms of the motion and interactions of their microscopic constituents. An example of a principle theory is thermodynamics, and an example of constructive theory is statistical mechanics, which reduces the behavior of gases to the motion of atoms. Here is how Yuri Balashov and Michel Janssen describe the two types of theories:"In a theory of principle […] one explains the phenomena by showing that they necessarily occur in a world in accordance with the postulates. Whereas theories of principle are about the *phenomena*, constructive theories aim to get at the underlying *reality*. In a constructive theory one proposes a (set of) model(s) for some part of physical reality […]. One explains the phenomena by showing that the theory provides a model that gives an empirically adequate description of the salient features of reality."[80]

Einstein introduced this distinction when discussing his 1905 theory of relativity, which he regarded as a principle theory, as it was formulated in terms of the two principles of equivalence of inertial frames for all physical laws, and constancy of the velocity of light (in vacuum for all inertial frames). This theory explains relativistic effects (such as length contraction and time dilation) as the physical phenomena compatible with the theory's principles. By contrast, Lorentz's theory (1909) derives the relativistic transformations and the relativistic effects from the electromagnetic properties of the ether and its interactions with matter, and because of this is a constructive theory. Again in the worlds of Balashov and Janssen: "Consider the phenomenon of length contraction. Understood purely as a theory of principle, SR explains this phenomenon if it can be shown that the phenomenon necessarily occurs in any world that is in accordance with the relativity postulate and the light postulate. By its very nature such a theory-of-principle explanation will have nothing to say about the reality behind the phenomenon. A constructive version of the theory, by contrast, explains length contraction if the theory provides an empirically adequate model of the relevant features of a world in accordance with the two postulates. Such constructive-theory explanations do tell us how to conceive of the reality behind the phenomenon."

Einstein introduced the principle/constructive distinction to express *his own dissatisfaction* for the theory of special relativity at the time. However, he could have said something similar for quantum theory. In fact, his preference for constructive theories is compatible with his idea that quantum mechanics is incomplete. Moreover, it fits well with his statistical interpretation of quantum theory, as it is a principle theory by constraining the phenomena with suitable rules. Einstein thought that kinematic theories are typically employed when dynamical theories are either unavailable or too difficult to build. For according to Einstein, "when we say we have succeeded in understanding a group of natural processes, we invariably mean that a constructive theory has been found which covers the processes in question."[81] As we have seen in section 2.3.1, his complaint was that we may never be able to complete quantum theory as o have a complete and constructive description. Some others have followed arguing that dynamical theories are more explanatory because in contrast with kinematic theories they provide insight of the reality underlying the phenomena. [82] That is, regularities and constraints over the possible experimental findings lack explanatory power because we still do not know why these constraints obtain. A reason is provided only by a dynamical, constructive theory, in which one is told the mechanism, or the microscopic story, that gives rise to the observed

[80] Balashov, and Janssen (2003).
[81] Brown, and Timpson (2006).
[82] See also Brown (2005); Brown and Pooley (2004).

behavior. For instance, it has been argued that the economy of thermodynamic reasoning is trumped by the insight that statistical mechanics provides. [83]

In the next section I elaborate as of how in the quantum domain radical relaxed realists like the IT approach favour principle theories. Then I show that bare GRW is not a constructive theory.

5.2. GRW as a Constructive Theory: GRWm

Quantum theory with the collapse postulate, as wells as the precisification given by the spontaneous localization theory, can be seen as principle theories. Indeed, the IT approach explicitly accepts this by claiming that to explain is to containing the phenomena without the need of a dynamical account. More specifically, Hilbert space is thought as a "the kinematic framework for the physics of an indeterministic universe, just as Minkowski space-time provides the kinematic framework for the physics of a non-Newtonian, relativistic universe."[84] Indeed Bub and Pitowsky argue that we should favor explanations given in terms of principle theories: "there is no deeper explanation for the quantum phenomena of interference and entanglement than that provided by the structure of Hilbert space, just as there is no deeper explanation for the relativistic phenomena of Lorentz contraction and time dilation than that provided by the structure of Minkowski space-time."

Along the way Einstein thought, one could instead think of quantum theory as a phenomenological theory. Since the spontaneous localization theory makes the quantum evolution precise, then one can think of it in the same way. That is, the spontaneous localization theory can be thought as the *quantum analog of thermodynamics*: just like thermodynamics gives us information about the kinds of regularities that the macroscopic phenomena obey, so the quantum recipe, now precise, is giving us information about the macroscopic world by specifying the statistics of the experimental results are supposed to conform to. In both cases one is agnostic about the microscopic state of affairs that would explain why such regularities obtain, and where these empirical results are coming from. Based on the success of the derivation of laws of thermodynamics from the microscopic description provided by classical mechanics, people like Einstein have hoped to provide a similar understanding of the quantum rules. Presumably, when Einstein and Schrödinger were asking for a completion of quantum theory they were asking for something analog to statistical mechanics for thermodynamics. In other words, by applying statistical methods one would arguably derive, under suitable conditions, the laws of thermodynamics in terms of the microscopic description in terms of the interactions of classical particles. Similarly, the Born rule should be understood as a macroscopic law derivable in terms of a more fundamental microscopic description. This is the sense in which I think that robust realists like Einstein or Schrödinger would want more than merely recover the macroscopic descriptions. That is, the robust realist is likely to look for a microscopic description that would dynamically explain, by describing the motion and the

[83] Brown and Timpson (2006).

[84] Bub and Pitowsky (2010).

interactions among the microscopic constituents, why macroscopic superpositions are not observed and why the quantum rules obtain. The pilot-wave theory can be arguably considered to provide such a microscopic understanding of the quantum rules: the microscopic three-dimensional particles the theory postulates combine and interact to form macroscopic objects. In contrast, this is not the case for the spontaneous localization theory. The description provided by the wavefunction is strictly speaking not microscopic, and it is not there-dimensional. This is the result of reading a theory developed to make precise an instrumentalist theory without turning it into a robust realist one: now we know precisely when to apply the rules without knowing why they hold, as only a robust theory would tell us that.

Arguably Ghirardi himself was a robust realist *a-la* Schrödinger and Einstein and liked a constructive understanding. Presumably, it is because of these robust realist tendencies that he argued that his theory is not a theory about the wavefunction, as we saw in section 2.3.2. So, after he developed a precise quantum recipe with which any instrumentalist should be content, he wanted to provide a microscopic quantum description from which the quantum recipe could be understood dynamically and constructively. Only this non-bare theory becomes a dynamic, constructive theory like the pilot-wave theory. His theory was dubbed GRWm. As we have seen, in this theory physical objects are not described by the wavefunction but rather by a three-dimensional matter density field defined by suitably integrating the squared norm of the wavefunction. This approach is similar to the one attempted by Schrödinger to complete quantum theory as described in section 2.3, with the difference that here the wavefunction spontaneously localizes. Arguably, by providing such a microscopic picture of reality, a dynamic, constructive explanation is possible. [85]

5.3. Bare GRW as a Non-constructive, Dynamical Theory?

To come back to the discussion in the previous sections one has the following correspondences: a robust realist like Einstein would favor constructive theories, would regard quantum theory to be incomplete, and thus would like to solve the 'big' measurement problem; a radical relaxed realist (like Bub and Pitowsky) would instead favor principle theories, and would regard the realism problem as the 'small' measurement problem. What about the non-radical relaxed realist? That is, what about bare GRW? Is bare GRW, the view that the spontaneous localization theory is a theory about the wavefunction, a principle theory too, just like the original spontaneous localization theory? One the one hand the proponents of wavefunction realism claim they are robust realists, as they wish to find an understanding of the fundamental reality. We have already disputed that, but let's assume that's true. If so, and if the parallel I have drawn here is correct, the theory they propose should be constructive. As we have seen, a constructive theory would require a particular type of derivation of the quantum rules (the principles) in terms of the fundamental (microscopic and three-dimensional) ontology. That is, constructive theories are dynamical, in contrast with principle theories. This is what happens in

[85] Similar considerations apply to the already mentioned GRWf.

the derivation of the laws of thermodynamics in terms of the Newtonian dynamics, as we noted already, and in the pilot-wave theory. On the other hand, the explanation of the macroscopic phenomena in bare GRW is instead different, as the theory is fundamentally non-constructive in nature, as the explanation they provide is not dynamical. In fact, wavefunction realists wish to derive macroscopic properties from the quantum rules and the wavefunction. As we have seen, Albert uses functionalism and Ney uses symmetries to establish the derivative reality of three-dimensional objects. In this way they provide principles that would need to constrain the phenomena: Albert constrains them by postulating what the Hamiltonian needs to be, and Ney constrains them by postulating that certain symmetry properties should hold. Notice, parenthetically, that this correlation between wavefunction realism and non-constructive principle theories holds true for any quantum theory with the wavefunction alone, like the many-worlds theory: in fact what prevents any such theory to be a constructive theory is not that the wavefunction is a wave, but rather that it is a wave in configuration space and not in three-dimensional space.

However, bare GRW is not a principle theory either. In fact, there is a sense in whicg the defender of bare GRW care about the dynamics, in contrast with what happens for the proponents of principle theories. In fact principle theories aim at systematizing the phenomena, regardless of the underlying dynamics. Instead, the case of bare GRW is different. First, the precision problem motivated to have a precise equation for the dynamics at all scales (indeed, the original paper of GRW is called "Unified Dynamics for Microscopic and Macroscopic Systems"). Moreover, as we have seen, Albert's functionalist account uses the dynamics even if not constructively, using the *form* of the Hamiltonian (rather than its solutions) to 'recover' three-dimensional objects.

These two features, having a non-constructive explanation and giving importance to the dynamics, seem to pull in opposite directions: the former pushes towards radical relaxed realism, while the latter towards robust realism, making bare GRW a peculiar hybrid of the two positions. The following table summarizes this conclusion.

Table 2:

Type of Realism	**Type of Problem**	**Acceptable Theory**	**Type of Explanation**
Robust	Completeness	Hidden variable type	Constructive/dynamical
Non-radical Relaxed	Precision	Hidden variables, modified Schrödinger dynamics, many-worlds	Non-constructive/dynamical
Radical Relaxed	Adequacy	von Neumann rule	Principle/kinematic

6. Two Dilemmas for the Wavefunction Realist

One could argue that the robust version of realism is the one which truly capture the spirit of scientific realism, while the relaxed version is too weak: a true realist is someone who is interested in the microscopic reality behind the phenomena the interaction of which is able to dynamically explain them. Such a robust realist will be concerned with a microscopic dynamical understanding, and she will look for constructive theories which, in the quantum domain, supplement the description provide by the wavefunction like the pilot-wave theory. If so, then the robust realist puts too much focus on the measurement problem as usually understood, namely as the problem of precisely eliminating the macroscopic superpositions. A robust realist should think that quantum mechanics is just like thermodynamics, and she should look for its corresponding dynamical, microscopic explanation. Two solutions of the precision problem, namely the spontaneous localization theory and the many-worlds theory, do not provide a dynamical explanation but merely a set of principles to constraint the phenomena. As such, these theories should not have been considered by the robust realist in the first place.

Alternatively, one could maintain that a realist could look for a fundamental (rather than microscopic) reality beyond the phenomena. This is what the wavefunction realists provides, and it works well especially in the case of the spontaneous localization theory, which they interpret as bare GRW theory. However, I have argued that wavefunction realism provides a hybrid view in which the dynamics is important, as for the robust realist, but the explanation is not constructive, as the relaxed realist. This peculiar hybrid is created by taking a principle theory (spontaneous localization theory) and reading it at face value (bare GRW) without thinking of it as a constructive theory.

Because of this, the wavefunction realist finds herself to face two connected challenges, the first of which has been outlined in section 5.3 and which can be summarized as follows. The wavefunction realist does not see herself as a relaxed realist: in her framework the wavefunction is not supposed to prescribe how to restrict the phenomena but rather it is supposed to characterize the fundamental ontology. For this reason, I think there is a tension in the wavefunction realist framework, namely the one between her desire of robust realism, and the kind of explanation the theory provides, which is not in constructive terms. The defender of bare GRW starts off as a robust realist but she ends up too close to the IT approach that she presumably would have liked. If so, however, one may wonder what the point of solving the measurement problem is, if one can do without it, as the IT approach does. Arguably, the point is the problem of precision: we want a precise rule to specify the wavefunction collapse. But why would one want a precise rule if she cares only about the appearances? It seems to me that one would care about the precision of the rule ultimately if she cares about the dynamics: it is because one wishes a *unified dynamics which is applicable at all scales* that one is interested on theories like the spontaneous localization theory. However, why are wavefunction realists interested in the dynamics, if they provide a non-dynamical explanation?

Moreover, there is another tension in the wavefunction realism framework between their realist desire to read bare GRW as a theory about the wavefunction, and their desire of understanding the macroscopic phenomena in terms of the microscopic ones, as it is done in statistical mechanics, in a constructive way. In other words: if someone wishes quantum theories such as the spontaneous localization theory to be about the wavefunction only, then she should not be too attached to a constructive understanding. However, Albert,[86] a prominent wavefunction realist and supporter of bare GRW, has defended the statistical mechanics (constructive) explanation of thermodynamics. In addition, he argued that the same explanatory strategy used in the classical domain would extend in the quantum domain, and that bare GRW, because it is a probabilistic theory, would be better than other deterministic theories. The reasoning goes roughly as follows. The Boltzmannian approach to statistical mechanics is one in which one suitably derives the macroscopic laws of thermodynamics from the microscopic classical dynamics. In particular, one can derive in probabilistic terms the second law of thermodynamics, which states that entropy (almost always) increases. To do so, one needs three ingredients: the dynamics, the statistical postulate and the past hypothesis. The past hypothesis assumes that the universe started with a state with small entropy, while the statistical postulate establishes that each possible state of affairs has the same probability of happening (and makes sense of the probabilistic understanding of the second law according to which it's overwhelmingly probable that entropy will increase). Albert argues that in the context of a theory such as bare GRW, in which one has the probabilities already in the dynamics, one could dispense of the statistical postulate. Because of this reason, bare GRW would provide an understanding of the laws of thermodynamics by relying only on two principles. As a result, bare GRW should be preferred to its deterministic alternatives, as the pilot-wave theory, which still would need three ingredients. However, the explanation given by bare GRW is a non-constructive explanation, while the quantum statistical explanation is constructive. How are they compatible? Albert would say that first shadows of the wavefunction would form, and then they behave as microscopic there-dimensional objects to which we apply the statistical mechanical machinery as in the classical example. However, there are two types of explanations: the formation of the shadows, which is non-constructive, and the constructive statistical mechanical explanations. Because of this, the theory seems less simple that the pilot-wave theory, in which all explanations are constructive. One could insisted that bare GRW is still simpler because there are less postulate However, it has been argued that the statistical postulate is not needed in the pilot-wave theory too.[87] If so, then any advantage of bare GRW seems to evaporate, and the tension between the two types of explanations remains.

[86] Albert (2000).
[87] Allori (2020b).

7. Conclusion

I have argued in this paper that, contrarily of the common understanding, it is not obvious what the problem of making quantum mechanics amenable to a realist interpretation is supposed to be. While usually realists have identified the problem to solve with the measurement problem, namely the problem of eliminating macroscopic superpositions in a precise manner (what I have called the problem of precision), I have argued that a 'robust' realist, interested in a dynamical and constructive understanding of the phenomena, should instead be interested only in a particular way of solving the measurement problem, namely in theories which also solve the problem of completeness. Instead, theories like bare GRW, namely the spontaneous localization theory understood as a theory about the behavior of the wavefunction, which only solves the measurement problem by (precisely) eliminating macroscopic superpositions without completing quantum theory, provides a non-constructive explanation. I have also argued that the non-constructive understanding of bare GRW combined with the importance given to the dynamics creates two tensions. First, it is a realist theory only in a relaxed sense. It is not an instrumentalist theory, but it is not robustly realist either: in fact it seems more similar to approaches like the IT interpretation, in which one merely cares about systematizing the phenomena, than many defenders of the theory may like. Second, it is unclear how to reconcile this non-constructive explanation with the constructive explanation in statistical mechanics, if one wishes to argue that the explanation of the laws of thermodynamics should not substantially change when moving from the classical to the quantum domain.

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