Some Simple Questions and Answers about Bohmian Mechanics

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What is Bohmian mechanics?

Bohmian mechanics is a deterministic theory according to which everything is made of particles, just like in classical physics, but they move along non-classical trajectories. This theory accounts for the observed quantum effects such as interreference and diffraction patterns in particles experiments. It was developed starting from the work of Louis de Broglie in the 1920s and by David Bohm in the 1950s. The particle trajectories are determined by a first-order equation, called the guidance equation, which contains the same wave function we find in quantum mechanics, namely one which evolves according to the Schrödinger equation.

Note: sometime people call a theory a 'Bohmian theory,' where the theory has a clear (meaning, explicitly stated) three-dimensional, microscopic ontology, whose evolution is either deterministic or stochastic, and determined by a wave function, which does not necessarily evolve according to the Schrödinger equation. The ontology can either be particles or fields, but not something in a high dimensional space (like the wavefunction). This is I think somewhat confusing, as the GRW theory, where the wave function evolves stochastically according to an equation which is different from the Schrödinger equation, can be seen as a Bohmian theory when seen as theory about the behavior of a three-dimensional matter field, or particles, or even a four-dimensional event ontology (the 'flashes').

What is its relationship with quantum mechanics? Why does someone need another theory beyond quantum mechanics?

Because quantum mechanics is either empirically inadequate (if understood as a theory about a Schrödinger-evolving wave function) or imprecise, artificial, or even incomplete (when introducing the collapse rule to make it adequate empirically). Let me explain.

Quantum mechanics is a theory which has as a fundamental equation, the Schrödinger equation, which is an equation for an object called 'the wave function.' However, since the Schrödinger equation is a wave equation, and waves superimpose (think of like water waves), superpositions for, say, electrons, namely superpositions at the microscopic level, will propagate at the macroscopic level into superpositions of stuff we can directly observe. For instance, take a cat in a box with a vial of poison hooked up with a radioactive substance so that if the radioactive substance decays, the vial is broken, and the poison released killing the cat (this is the infamous Schrödinger cat experiment). So, in essence, we can use the cat as a living detector to see whether the substance has decayed or not. Nonetheless, the radioactive substance may be in the superposition of a decayed-and-not decayed substance, because the Schrödinger equation is linear. If that's the case, then this superposition will propagate to the superposition of a dead-and-alive cat. However, when we check on the cat, we never find her in such a superposition state: she's either dead or alive. That is, our measurement device always gives us a result: dead-cat (meaning, decayed nucleus) or alive cat (meaning undecayed nucleus). The macroscopic superpositions predicted by quantum theory are never observed, so the theory is empirically inadequate.

To fix this, we need to postulate that every time there is a measurement of some sort (every time we expect not to find macroscopic superpositions), the wave function randomly collapses into one of the terms of the superpositions. So essentially, when we open the box and check on the cat, we measure whether the radioactive substance has decayed, and we may well be killing the cat.

What's wrong with this is, except murder of course, is that we do not really know what is meant by measurement here: aren't observers and measurement apparatuses also physical entities which should obey the Schrödinger equation? Does this have to do somehow with our role as conscious observers? This would be a peculiar state of affairs because it would make it impossible to construct a quantum theory of the universe, because who's observing the universe and collapsing its wave function? Other people have instead argued that it only seems that the wave function randomly collapses, when instead what happens is not a real physical process. The wave function represents our information of the system, not the system itself. So, when the cat is in the box, we do not know whether she's alive or dead, but when we open the box and look, then we gain new information, and we update the wave function accordingly. However, the natural question in this case is how our knowledge of the system manages to give rise to phenomena like interference, which are objective. They would not disappear if we knew everything, don't they?

So, quantum mechanics is either inadequate or incomplete, so it needs fixing. Bohmian mechanics takes what's good in quantum theory (the Schrödinger equation) and removes its questionable assumptions (consciousness collapsing the wave function, measurement apparatuses being fundamental, and the like) and provides a microscopic description of reality in which the phenomena are described in terms of particles moving around deterministically. No need of consciousness killing cats, no mysterious terminology in the formulation of the theory. Simply a deterministic theory of particles following trajectories. As one of my Bohmian friends always say, it's the obvious ontology moving in the obvious way.

OK, but how does it solve the problems if quantum mechanics? Does the wave function collapse?

In Bohmian mechanics the wave function which is Schrödinger-evolving is the wave function of the universe Ψ (capital psi). Nonetheless, we can talk about wave function of more 'regular' systems (like a cat, or a table, or an electron, made of particles with configuration x) by plugging in the universal wave function the actual configurations of everything else (Y), except the system. This is the system's *conditional wave function*: ψ(x)= Ψ (x,Y). One can prove that, when the system is suitably decoupled from its environment, also the conditional wave function (in addition to the universal wave function) obeys the Schrödinger equation. Instead (in contrast with the universal wave function which always evolves according to the Schrödinger dynamics), when the system interacts in a 'measurement'-like situation the conditional wave function randomly collapses like in quantum mechanics. That is: the universal wave function evolves according to the Schrödinger equation, while the conditional wave function generally does not. In other words, it's not two incompatible evolution equations for the same object, like in quantum mechanics, but two evolution equations for two compatible objects - the universal and the conditional wave function.

How does Bohmian Mechanics save the cat (solves the measurement problem)?

In Bohmian mechanics the cat is made of particles, like everything else. To say that the cat is alive is to say that her particles move and interact with one another as to produce those chemical reactions typical of a living being. The same for a dead cat, with the suitable adjustment. The wave function of the cat governs or guides the motion of the particles, and thus there is an obvious sense in which one can think of the component of the wave function of a living cat and of a dead cat: the one of the living cat will be such that the particles whose motion it governs are moving and interacting with one another in a 'living' manner. Since the wave function evolves according to the Schrödinger equation, it can be in a superposition of living and dead wave functions, but the particles NEVER are: they are either 'under' one component of the wave function or the other, and thus they will be effectively either guided by a living cat wave function or by the dead cat one. That is, the cat is at all times, either alive or dead, regardless of whether the wave function is in superposition or not.

Is it correct to say that in Bohmian mechanics there are particles and waves?

I do not think so. If someone says that, one should not take her too literally. In fact, there is a pictorial sense in which this is true, but a deeper sense in which it is not. Sometimes Bohmian mechanics is presented in this way: there are particles, evolving according to a guidance equation which is determined by the wave function, and the wave function, evolving according to the Schrödinger equation. So, in a sense, there are two things. For this reason, sometimes the theory is called pilot-wave theory. There are particles, and the wave 'pushes' the particles around. This explains why we see interference in the two-slit experiment with particles: because the particles are 'pushed' by the wave along trajectories which aren't classical. However, this presentation suggests very strongly that the wave function can be though of in terms of a physical field, much like the electromagnetic fields in classical electrodynamics. However, there is a very important disanalogy: while electromagnetic fields are fields in three-dimensional, physical, space, the wave function is actually a field in a much higher dimensional space. In fact the wave function is a function of all particles configurations, so it is a function of 3 times N variables (one for each component x,y,z), if there are N particles in the universe. So, if one wishes to insist on the wave-pushing-particles picture, then one has to recognize that this wave does not even live in the same space as the particles! The 'pushing' of the wave function, I think, is better understood as law-like. That is, as we can think of gravity as pulling objects towards the ground, we can think of the wave function as pushing the particles around. In other words, I think that the best way of thinking of the wave function is as an ingredient in the laws of nature, something necessary that we need to have in order to recover the experimental data, rather than thinking of the wave function as representing a field in a high dimensional space.

Is Bohmian Mechanics a hidden variable theory?

Sometimes the idea behind Bohmian mechanics is expressed by saying that it is the rational completion of quantum theory, meaning that the reason why quantum theory is so weird is because we do not have the whole picture, which instead is given by Bohmian mechanics. Usually people say that while in quantum mechanics the complete description of the system is given by the Schrödinger-evolving wave function, Bohmian mechanics adds to this the particles' configuration, thereby supplementing the quantum mechanical description. This is what people have in mind when they call this theory a 'hidden variable' theory: there is something, namely the particles' positions, which quantum theory does not specify, and because of this provides only an incomplete picture of the world. However, I do not think it is the best name for this theory, as it suggests that these hidden variables are mysterious, which is not the case: they are hidden only in the sense that they are not present in the textbook theory. The particles are indeed what Bohmian mechanics is about, what we directly observe in experiments is position of stuff. Indeed, one could say the opposite: the wave function is hidden, positions are not.

Wait! Didn't von Neumann prove that hidden variables are impossible?

This is what has been believed for a very long time, and which essentially has halted the research in this field for way too long. Von Neumann's theorem did prove something, but not that we cannot make sense of quantum mechanics suitably completing its description.

His proof is by contradiction: assume the opposite what you are trying to prove (hidden variables are impossible) and show it leads to contradiction. So, he considered a theory with included both quantum mechanics and some hidden variables, and the he proved that, under some reasonable assumptions, a theory like that would lead to some mathematically impossible relations (such as 5 is greater than 7… not really but it gives you an idea). So that the only way out is to deny the possibility of deterministically completing quantum mechanics with hidden variables. Schematically, the structure of the argument is, allegedly, as follows: assume hidden variables, show it leads to contradiction, therefore no hidden variables, we cannot do better than quantum mechanics.

What's wrong with this theorem? First of all, the reasonable assumptions were not really reasonable. But that's not the main thing wrong with it, as other theorems in a similar vein not relying on these questionable assumptions have been proposed later. So, how are these theorems compatible with the existence of Bohmian mechanics, which is a deterministic completion of quantum mechanics? What's wrong with these new theorems? What's wrong is the hidden assumption, not the hidden variable. In his proof, von Neumann assumed, like it's common to do in quantum mechanics, that some mathematical objects called self-adjoint operators represent properties of a system which could be measured in a given experiment. Say we want to know the energy of a system. Quantum mechanics tells you that the possible values for energy of the system are given by the eigenvalues of suitable self-adjoint operator, in this case the Hamiltonian. When we perform an experiment to measure the system's energy, the result we get is taken to be the energy of the system before the experiment, which the experiment reveals to us. However, this operators-as-property assumption is not reasonable, because it implicitly assumes that making an experiment on a system does not significantly change it. Assuming this is fine classically: when I think I have a fever and I measure my temperature with the thermometer, I am actually interacting with the thermometer and both of us somehow change. What the thermometer shows when I read the temperature is NOT my initial temperature. Rather, it is the equilibrium temperature reached by the system composed by me and the thermometer. The liquid in the thermometer is such that the equilibrium temperature is so close to my temperature before I measured it that we can forget about all the me-interacting-with-the-thermometer business. But it's not always like this. Sometimes in order to find out information about the system, we need to perturb it so much with the experimental apparatus that what we get as result cannot really be considered as a feature of the system before the experiment. Rather it tells us something about the interaction between the system and the apparatus. For instance, I want to know the position of this table. I need to switch on the light to see it. The photons bounce off the table surface into my retina, the table recoils a little bit but in a negligible way, and what I see is approximately the table position before I switched on the lights. Now I want to know the position of an electron. The photon will hit it, and they both will scatter around. The recoil of the electron is no longer negligible, and the photon I see do not tell me much about the electron position before I switched on the light because it actually contributed to change it. That is, in general the interaction of the system and the experimental apparatus will be sufficient to disrupt the system so much that the experimental results are not the values of the properties before the experiment.

Therefore, von Neumann's theorem, and the others after it, does not prove that hidden variables are impossible, but rather it proves that experiments may often reveal nothing about the system. In other words, many experiments are not measurements.

This somehow reminds me of the Heisenberg principle…

One formulation of the Heisenberg principle says that we cannot know at the same time the exact position and the exact velocity of a particle. That is, it is a principle that tells us that there are limits about what we can know about a system. What we can know (the information we can have access to) has to do with how the system we are investigating correlates with another system that we can access directly because macroscopic (like a measurement apparatus). This is what we do when we want to measure some property: we hook up the system to some other calibrated system to read from it the property we want to measure. For instance, if I want to know how much this book weights, I put it on a (properly calibrated) scale. The book interacts with the scale, whose pointer moves to indicate a location labelled: '1 Kg.' Because of this, we acquire the information that the book on the scale weights 1 Kg. However, the system-apparatus interaction has to be just right. As we have seen earlier, the system-apparatus interaction needs not disrupt the system too much, otherwise it's not a genuine measurement. That is, on one hand, we need the interaction to be not too strong. On the other hand, if a system and the apparatus do not interact at all, obviously there can be no system-apparatus correlation, and thus no knowledge of the system can be acquired by investigating the apparatus. So, for sure, we would like to have some interaction, and in order to have any at all, we need to be in non-equilibrium. In fact, when the system and the apparatus (or in general two systems) are in equilibrium, then nothing changes anymore. Knowledge needs interaction, and interaction, needs non-equilibrium.

We currently are in thermal non-equilibrium: coffee gets cold, ice melts, we get old, the universe keeps changing, entropy increases, and so on. However, in a Bohmian world, the configurations of the particles are in 'quantum' equilibrium with respect to the wave function. So, as in statistical mechanics I cannot extract anything from a system with which I am in thermodynamic equilibrium, there is only so much I can know about a system in quantum equilibrium. All we can know about particles locations is given by the system wavefunction, because that is the one which is not in equilibrium. That is, the maximal information we can have of where the particles are located is given by their wave function, which is the reason why it can be called the state of the system. This limitation of what we can know about a system is the core of Heisenberg's uncertainty principle: there is absolute uncertainty about what we can know.

Quantum equilibrium? What is that?

Technically, quantum equilibrium is the claim that positions of the particles in a system with conditional wave function ψ are distributed according to the square of ψ. It can actually be proven (with techniques similar to those used in Boltzmannian statistical mechanics) that for a typical Bohmian universe, namely for most Bohmian universes (as defined by the measure defined by $|\Psi|^2$), positions are ψ-squared distributed.

Given a typical Bohmian universe is in quantum equilibrium, we can be confident we are in one, and as a consequence of this one cannot have more information about the configuration of a system with conditional wave function ψ than what is given by $|\psi|^2$. In fact, the information about the system must be stored, somehow, in the configuration of the rest of the universe. But this is already taken into account in the conditional wave function (which is the universal wave function with the actual configuration of everything except the system in it).

Why does Bohmian mechanics have probabilistic predictions, if it is deterministic?

This is a legitimate puzzle: if Bohmian mechanics is a deterministic theory, shouldn't we be able to predict, like we do in classical mechanics, where the participles end up, and therefore get unique outcomes for experimental results? Instead, we get probabilistic distributions. Where are these probabilities coming from?

The short answer is: probabilities come from the same thing that explains the Heisenberg principle, namely quantum equilibrium.

The long answer is the following. Bohmian mechanics is a deterministic theory, and the particles trajectories are what they are, given the law and the initial conditions. But initial conditions are not accessible to us, because of quantum equilibrium, so any indeterminacy we see in the experimental results is coming from this uncertainty on initial conditions.

It can also be proven that if quantum equilibrium is true at one time, it is true at all times: if at a given time the particles are randomly arranged according to a distribution given by the square module of ψ , then they will continue to be distributed that way (this property is called 'equivariance').

That means that, when performing an experiment to determine where a particle is, the probability of finding it in a given place x is provided the square module of the conditional wave function evaluated at that point: $|\psi(x)|^2$. This is the Born rule of quantum mechanics. This is the formal proof that Bohmian mechanics makes the same predictions as quantum mechanics, as long as quantum mechanics makes precise predictions.

What do people mean when they say, in a derogatory way, that Bohmian mechanics is contextual?

A theory is said to be contextual when the value of a property depends on the way in which this property is measured. They depend on the context in which they're measured. This is a crazy feature, because we think of properties as characteristics of the object which should not change based on the context! Suppose I want to measure how tall you are. I set up an experiment to measure how tall you are and how much you weight. As a result, I get 6 feet. Then I set up another experiment, where I measure how tall you are and the color of your eyes. I do not expect to get a result different from 6 feet, because that's how tall you are. But if height is contextual, then I would! So, if Bohmian mechanics were contextual, then if would be extremely weird, to say the least, *ergo* the derogatory flavor of the remark.

People think that Bohmian mechanics is contextual because of theorems like the one of von Neumann: if we try to complete quantum theory with hidden variables, and we assume that operators represent properties, then we end up in situations in which we have to assume that these properties are contextual. However, as in the case of von Neumann's theorem, the mistaken assumption is the operator-as-properties assumption: sometimes experiments do not measure any properties. Rather, the results merely express the way in which the system interact with the apparatus and, depending on the experimental setting, the interaction is different.

Why should I believe that Bohmian mechanics is true?

Let me put it this way: If you love legos, you will love Bohmian mechanics 😉

Less cryptically, Bohmian mechanics makes sense, in contrast with quantum theory.

It makes sense mathematically: one evolution equation for the particles, one for the wave function of the universe, the rest (everything!) follows. No mysterious double evolution for the wave function, no assumptions on observers, measurements or the like: one can define the wave function of a system from the universal wave function as the conditional wave function, and one can show that it evolves according to the Schrödinger equation when the system can be thought of isolated, and it collapses in measurement-like situations. There are no postulates about how to obtain measurement results, no operators as observables assumption: operators are useful tools to describe the statistical distributions of the experimental results, which may or may not represent properties, depending on whether the experiment is or is not a genuine measurement.

It makes sense physically: stuff is made of particles evolving according to non-classical trajectories. This grounds a lego-bricks explanatory schema very similar to the classical way of understanding the world: the world is made of particles, which are the building blocks of everything else.

It is the simplest and most explanatory way of understating the quantum formalism in a way to reproduce the experimental data.

What are the challenges of Bohmian mechanics?

Sometimes, people complain that Bohmian mechanics is not testable because it makes the same predictions of quantum theory, and quantum theory has been developed first. This is certainly true, at least as long as the quantum predictions are unambiguous.

However, it's not even clear that quantum theory came first: Heisenberg's matrix mechanics was proposed in 1926, so did Schrödinger's equation, and the axiomatic formalization was proposed in 1932 by von Neumann, while de Broglie's idea of particle-wave association was proposed in 1923. So, arguably Bohmian mechanics came first.

Be that as it may, it is unclear why one should think that a given piece of evidence confirms quantum mechanics and not Bohmain mechanics simply because quantum mechanics was proposed first. Evidence is evidence, and it either confirm a theory or it does not, independently in whether a theory was proposed before or after another theory. So, it's better to say instead that the data confirm both theories, and there is no way of empirically distinguish between Bohmian mechanics and quantum theory (however, see later).

One could reply that when we have two theories that are empirically indistinguishable, then if one is simpler than another, we have reasons to believe the simpler theory. Quantum theory is simpler, as it has only one equation, the one for the wave function, so we should believe quantum theory over Bohmian mechanics. However, it is essential to remember that quantum theory is not even a theory: it talks about electrons, and protons, and stuff, but it is not clear what they are in the formalism. Even being charitable and granting that quantum theory is a theory, it may even have only one equation, but it is a complete mess in all other respects. It is not even a complete picture. So, would you rather read a book with all pages (Bohmian mechanics) or a book with half the pages (quantum theory) only because it's thinner?

Moreover, while it is true that quantum theory will make the same predictions as Bohmian mechanics, sometimes the degree of confirmation evidence provides for each theory is not the same. For instance, there is some experimental evidence of the Bohmian trajectories: weak measurements of velocities for photons in two slit experiments show trajectories compatible with the ones predicted by Bohmian mechanics. That is, you look at the result of these measurements, and you see non-classical trajectories. Since it is a theorem that Bohmian mechanics and quantum mechanics make the same predictions, such trajectories can be accounted also by quantum mechanics, a theory in which however there are no particles and no trajectories. What theory is this evidence confirming more? Which theory should I believe? One in which there are no particles and no trajectories (quantum mechanics) but the results somehow conspire as to display trajectories, or a theory of particles moving according to trajectories which looks like the one observed (Bohmian mechanics)? (One may argue that they're photon trajectories, and Bohmian mechanics works well only with massive particles. Fair enough. But it's not impossible to perform the same experiments with massive particles. So, let's wait and see.)

Furthermore, it is true that Bohmian mechanics and quantum mechanics make the same predictions, but only when the predictions of quantum mechanics are precise and unambiguous. There are cases in which quantum mechanics does not have the means of making precise predictions, while Bohmian mechanics does. There are several such examples, one of which is the case of time of flight experiments. These experiments measure the time for a particle to hit a screen. As anticipated, quantum mechanics prescribes that the possible experimental outcomes are given by the eigenvalues of a suitable self-adjoint operator. For some reason, quantum mechanics does not have a 'time' operator, so to compute the arrival times of particles, it needs to resort to classical approximations or other techniques. There are cases in which the results provided by these techniques differ from the results of Bohmian mechanics, so one could go out and measure which one is correct. So, if someone really cares about Bohmian mechanics being testable, there is a very important sense in which it is actually testable.

So, would you say that the real challenge of Bohmian mechanics is that it is nonlocal?

Bohmian mechanics is manifestly nonlocal. Whenever the wave function of a system is not a product of single-particle wave function, so whenever the wave function is entangled, the velocity of a particle will generically depend upon the positions of the other particles, regardless how distant they are. However, notice that this 'spooky action at a distance' cannot be used to send instantaneous messages. In fact, because of quantum equilibrium, we cannot control the particles positions, and so we cannot encode in them the information we wish to send.

Let me postpone for a second why nonlocality is a problem, and let's just ask whether this feature, namely nonlocality, is a feature distinctive of Bohmian mechanics or not. Think about quantum theory. The collapse rule, which randomly and instantaneously projects the superposition wave function in one of its components is manifestly nonlocal as well.

Moreover, Bell has shown that the predictions of standard quantum theory itself imply nonlocality. Einstein, Podolsky and Rosen (EPR) proposed an argument that if quantum theory were complete then it would violate a condition of locality. In fact, experiments on pairs of singlet-state particles sent in opposite directions would show perfect correlations which could only be explained by some sort of telepathy, or superluminal communication. Bell wanted to see whether it was possible to complete quantum mechanics avoiding this nonlocality and, by considering a generic completion of quantum theory by adding some 'hidden variables,' he arrived at his famous inequality. Since quantum mechanics instead predicts violations of such an inequality, one could test which theory is correct. When this test was performed, it showed that the quantum mechanical predictions are correct. That is, any theory which makes the same predictions of quantum theory has to be nonlocal, as locality was the only assumption made by EPR.

Notice: there are ways around this conclusion. For instance, there is another assumption in Bell's proof that Bell took for granted but could in principle be disputed: the so-called hypothesis of statistical independence, namely that the experimental settings do not depend on the distribution of the additional variables. One can reject this assumption at least in two ways. First, by allowing for the settings to depend on the hidden variables: these theories are called superdeteministic. Bell did not take them seriously because superdeterminism is the idea that everything is so working together that all systems are correlated with the choices of which measurements we perform on them.

Another way to get around statistical independence is to have retrocausal theories, in which the cause does not come before its effects. I cannot enter into the details here, but the question is this: people postulate superdeterminism or retrocausality to avoid nonlocality, but which assumption costs more? It is unclear to me that saving locality by assuming one of these conditions has more benefits than costs.

So, the bottom line is that: if this analysis is correct and nature is nonlocal, then it's not a problem of Bohmian mechanics that it is nonlocal. It is just what nature requires it to be.

All right, nature is nonlocal. Why is this a problem?

Nonlocality is a problem because it is in tension with relativity theory: there should not be any spooky action-at-a distance. So, if nature is nonlocal, how can we put together quantum theory (or better Bohmian mechanics) and relativity? People have constructed Bohmian theories which are relativistic invariant in the sense that they have a Lorentz invariant evolution for the wave function. However, they seem to require the existence of an undetectable preferred frame of reference (a 'foliation'), which fundamentally amounts to reintroduce a notion of absolute simultaneity. Be that as it may, this is the real challenge for the future, and it is a merit of the explicit nonlocality of Bohmian mechanics to have clearly shown where we should concentrate our efforts as physicists: to find a suitably relativistic quantum theory, given that nature is nonlocal.