Introduction

This book is a collection of essays devoted to exploring different aspects in the foundations and the philosophy of quantum theory. They range from issues about its compatibility of scientific realism to questions about the ontology of the theory; from questions about what is fundamental to questions about the nature of the wave function and of quantum objects. Accordingly, the book is divided into four parts, which however share some overlap in topics.

Part I: Realism

The first part deals with issues regarding scientific realism and quantum theories. Quantum mechanics is a fundamental physical theory. In virtue of this, scientific realists would think that it can inform us about the nature of the world. Nonetheless, for a very long time, many thought that quantum mechanics forced to anti-realism. Against the classical view that macroscopic objects and their properties can be understood as composed by microscopic objects governed by clear and precise laws, the Copenhagen school, championed by many of the founding fathers of quantum theory, promoted the idea that the quantum world is intrinsically incomprehensible and unknowable. All physics can do is predict experimental results, being unable to provide a satisfactory description of reality. That is, we should abandon the naïve idea behind scientific realism that theories can help us understand the world.

It is commonly accepted that the problem to solve to make quantum theory compatible with realism is the measurement problem. Quantum theory is a theory about an object, the wave function, which evolves in time according to an equation called the Schrödinger equation. It is a mathematical fact that, given that this equation is linear, sums of solutions are also solutions. That is, they also represent physically possible states of affairs. If quantum theory is complete, it describes both macroscopic and macroscopic bodies, and if microscopically one has a sum of solutions (a superposition), then this may propagate macroscopically. This is the typical case of the Schrödinger cat. Assume a cat is in a box where a radioactive atom is hooked up to a vial of poison so that if the atom decays the vial breaks and the cat dies. If the initial wave function is a superposition of the states “decayed” and “undecayed,” then the final state will be a superposition of “dead” and “alive” cat. However, we never observe such macroscopic superpositions: the cat is either dead or alive. In other words, if the wave function evolves according to the Schrödinger equation, and the theory is complete, there are unobserved macroscopic superpositions. If we want to think of quantum theory as compatible with realism, we have to fix this problem. This is traditionally taken care of in the standard formulation of quantum theory by the collapse rule: it is postulated that whenever there is a measurement (like us opening the box and observing the state of the cat), the wave function no longer evolves according to the Schrödinger equation but randomly collapses into one of the terms of the superpositions. However, this is unsatisfactory: what physical processes are to be considered measurements? While some have searched for a solution of this problem and connected others outside of quantum theory, others have argued that decoherence, roughly the interaction of a system with its environment, can help us solve them from within the theory: the system is effectively “measured” by its interaction with the environment, so that the superpositions effectively collapse into one of its terms. **Davide Romano** in Chap. 1 criticizes this perspective. One of the challenges is that quantum theory needs to provide a wave function for the original system after the interaction with the environment, starting from the wave function of the system composed of the original system and its environment. Some have argued that decoherence can provide us with such a wave function. Romano instead shows that this is not the case. Moreover, standard quantum theory has the problem of explaining why the wave function can be written as a function of different variables (position, momentum, etc.), but everyday objects seem to be located in space. It has been maintained that decoherence can explain why the position basis is privileged, but Romano maintains that the argument is circular, if applied to the standard theory: positions are introduced as a privileged in the system-environment interaction.

If we leave standard quantum mechanics aside, other solutions of the measurement problem are fully compatible with a realist interpretation: the pilot-wave theory (also known as de Broglie-Bohm theory, or Bohmian mechanics), the spontaneous collapse or spontaneous localization theory (which also goes under the name of dynamical reduction theory, or GRW theory), and the many-worlds theory (also popular under the name of Everettian mechanics). Roughly put, Bohmian mechanics is a theory in which the description of the system is provided by the wave function that is supplemented by particles’ positions; in the GRW theory, the wave function no longer evolves according to the Schrödinger equation to allow for the collapse to be built in the dynamics rather than postulated; Everettian mechanics instead takes seriously the idea that all possible terms of the superpositions are actual and realized in different “words” which no longer interact with one another.

Granting that our belief in scientific realism is justified, one natural question is how these theories connect the physics with the metaphysics. This is a matter of controversy. Physical theories use mathematics in their formulations; therefore, if these theories describe the world, one needs to identify a correspondence rule to identify which mathematical entities represent physical objects, their properties, and their laws of evolution. Within the classical domain, this correspondence was straightforward, because it was clear to Newton what his theory was supposed to be about (particles), and there was an obvious mathematical way of representing their nature (points). That is, Newton went from his metaphysical hypothesis to the mathematical formalism. Instead, in quantum mechanics the opposite is true: we have a mathematical formalism that works very well in predicting experimental results, that is, there is a correspondence between the formalism and the world at the macroscopic level, but we need to physically interpret it in the microscopic domain: what do the various variables mean, from the point of view of describing the nature of the world? Some variables that appear in the quantum formalism are the same as in the classical case, and thus they seem to have an obvious correspondence: t is time, m is mass, V or F represents the interaction. But interaction between what? There is an evolution equation for the wave function, so presumably this is the object representing what physical objects are made of. It would be nice if the wave function could be represented as an oscillation in three-dimensional space, just like electromagnetic fields. However, this is not possible, as the wave function, at best, can be interpreted as oscillating in a space with much higher dimension, the so-called configuration space. The view according to which the wave function in this high-dimensional space is the fundamental ontology has been dubbed in the literature as "wave function realism." If we take this route, then we have the problem of making sense of this connection given that the wave function lives in some abstract space, rather than three-dimensional space. Motivated in part from this problem, some have contended that the wave function is not the right kind of mathematical object to represent matter, only some kind of "local beables" can suitably do that. Roughly, a mathematical object is a be-able, as opposed to observable, in the sense that it represents something; the locality condition refers to the fact that it is in three-dimensional space rather than in a space of higher dimensions.

A particular way of implementing this view is the so-called "primitive ontology" framework. The idea is that the wave function should not be seen as representational but rather it should be thought as having another role in the theory. In particular, some have argued that the wave function is more similar to a law of nature or a property than to an object, but it is not straightforward to spell out what this is supposed to mean. On this basis, in Chap. 2, **Matthias Egg** critically engages with the primitive ontology approach, which he dubs "quantum fundamentalism." He argues that it is in tension with the basic idea of scientific realism if one pushes the view to the conclusion that the wavefunction is not real. In fact, while quantum fundamentalism seems to entail that we are more justified in believing in the entities which belong to the fundamental ontology rather than those which are not in it, scientific realism instead pushes in the opposite direction. That is, given we have stronger evidence from them, it encourages us to think that we are more justified in believing in nonfundamental objects than in fundamental ones.

As an alternative to the primitive ontology approach, we have seen, one finds wave function realism. A natural question is whether there are some principles we can use to determine which ontology should we choose for quantum theory. This is discussed by **Vera Matarese** in Chap. 3. She presents and critically analyzes two of the principles that have been presented in the literature: the dynamical matching principle and the minimal divergence norm. The former asserts that the fundamental structure of the world should match the structure of the dynamical laws of the theory and has been used to support wave function realism; the latter asserts that one should choose the theory that minimizes the difference between what the theory says the world is like and how the world appears, and it has been taken to support the primitive ontology approach. Matarese argues that this is not straightforward and that both principles can be used to support either view. Thus, she concludes, ultimately these principles cannot guide our choice of ontology but rather they are best regarded as useful tools to restrict the space of plausible ontologies.

In addition to the three theories discussed above, there are other proposals. For instance, the information-theoretic account maintains that quantum theory should be understood as providing constraints over possible experimental outcomes. It has been argued that this approach is compatible with realism because it provides a mind-independent, objective description of reality. **Laura Felline** critically engages with this approach in Chap. 4, by comparing it with Bohmian mechanics and the GRW theory. While the proponents of each approach argue that their framework is to be preferred, Felline instead maintains that, in a Kuhnian way, there is no common standard to which one could compare them with one another: they ask different questions, and provide different answers; they have different aims, and they provide different solutions. In this way, they both can provide us with different but equally important insights.

Within the information-theoretic framework, the wave function is taken to be epistemic, that is, it represents out knowledge of the system rather than the system itself: for instance, when we make a measurement on a system, the wave function changes because we acquire new information. It has been argued that there are strong reasons, such as the PBR theorem, which disqualify epistemic views of the wave function. Under this assumption that the wave function has to represent something real (it is ontic), in Chap. 5 **Travis Norsen** explores the question regarding the best way of thinking about the wave function. He first criticizes the various proposals, from wave function realism to the approach that it is a law of nature or a property, and he proposes to take more seriously the possibility of constructing a theory in which the information contained in the wave function is captured by some sort of orchestrated field, like it is (imperfectly) done in his theory of exclusively local beables.

One may think that all these controversies about the compatibility between realism and quantum theory is a waste of time because realism is doomed after all. If so, why should we care about the theories discussed, which take realism for granted? **Darrell P. Rowbottom** argues that even if one is not convinced that scientific realism is true, theories with a clear ontology may be useful regardless. In Chap. 6 he shows that also an instrumentalist should prefer a theory with a clear ontology, like Bohmian mechanics. In fact, even if the theory were to fail to be approximately true (for instance because the notion of approximate truth is not well defined, or because the theory is nonrelativistic), Bohmian mechanics possesses significant theoretical virtues. Rowbottom argues that they can be valuable, among other things, in the understanding of how phenomena are connected with one another. In this way, even if one does not endorse scientific realism, one can get the next best thing, namely understanding, by endorsing Bohmian mechanics.

Part II: Ontology

Setting the worries about realism aside or granting that there is value in having a clear model also from the point of view of the anti-realist, one can move to more specific questions about ontology. We already mentioned wave function realism, which regards the wave function as the fundamental ontology of the worlds, as well as the primitive ontology approach, which views quantum theories to be about a three-dimensional microscopic ontology rather than the wave function. Then we have space-time state realism, according to which the fundamental ontology of a quantum mechanical world consists of a state-valued field evolving in four-dimensional space-time. There are also many other approaches based on the notion of local beables, and the one dubbed "minimalist ontology," which seem to come from similar perspectives: the former maintains that the ontology of a theory needs to be local, and the latter argues for an ontology of point particles. Given these similarities, all these proposals have been grouped under the same category. However, they are different. **Andrea Oldofredi**, in Chap. 7, compares and contrasts them. Oldofredi underlies how some local beables, such as electromagnetic fields, may not be suitable primitive ontologies. This is so because the primitive ontology framework, in addition to having the requirement of a local (i.e., three-dimensional) ontology, also gives importance to symmetry properties, which would be violated if electromagnetic fields were included in the primitive ontology. In addition, it is noted how the primitive ontology program is more flexible than the minimalist ontology framework as the latter postulates a fundamental ontology of matter points, while the former leaves open the possibility for other types of ontologies such as matter fields or spatiotemporal events ("dubbed flashes").

This is evident in the framework of the various GRW theories. In fact, while Bohmian mechanics is easy to interpret as a theory of particles, in the primitive ontology framework one cannot really talk about "the" GRW theory, because in this theory there is only the wave function, and the wave function, not being a three-dimensional object, is not a suitable primitive ontology. Rather, different possible primitive ontologies for the same GRW wave function evolution have been proposed: a continuous matter density field, flashes, or even particles. To each primitive ontology corresponds a distinctive GRW-type theory: GRWm, GRWf, and GRWp, respectively. In Chap. 8, **Elizabeth Miller** challenges the viability of the matter field and the flash ontology. As Egg criticized the primitive ontology program from a scientific realist perspective, Miller argues that these ontological choices for the GRW theory undermine the motivation to endorse the primitive ontology view to start with. The argument is that if one resists a wave-function-only-GRW-theory because there are superpositions, these superpositions still remain in GRWm and GRWf, albeit in a subtle form which is worth investigating.

In the literature, the proponents of the primitive ontology program have always taken for granted that the primitive ontology had determinate values. Instead, **Cristian Mariani**, in Chap. 9, argues that in the context of GRWm it would be more appropriate to think of the primitive ontology as indeterminate in the sense of lacking definite properties. In fact, it is argued, this would allow us to make sense of the so-called “inaccessible mass” problem. In GRWm, the matter density function is many-to-one: two different wave functions describing physically different situations can generate the same matter density. In the literature, the locutions “accessible” and “inaccessible” have been introduced to describe the different physical situations represented by the same matter density. However, if the inaccessible matter density is to be taken ontologically seriously, Mariani argues that it then remains unclear what characterizes the two physically distinct situations. He therefore proposes that by thinking of the primitive ontology as indeterminate, one could still retain the proper reductive explanatory schema of the primitive approach.

Be that as it may, traditionally, the way in which scientific theories give us a picture of the world is spelled out in terms of specifying what the fundamental objects are, what their fundamental properties are, and what laws applies to them. However, there are departures from this general trend, some of which come as ways of responding to the so-called pessimistic meta induction argument against scientific realism. One of the strongest arguments for realism claims that if the theory is successfully reproducing the data, then we have grounds of believing it to be true. However, the pessimistic meta induction objection roughly states that empirical success cannot be an indication of truth since all past theories which have been successful turned out to be false. One of the responses to this challenge is structural realism, which urges us to rethink of our understanding about ontology. In fact, in this approach structure is all we are justified to believe, since structure has been preserved through theory change, and it is responsible of the theories’ success. There are various forms of structuralism, some of which have been motivated by features of quantum theory, such as entanglement: the fact that composite systems have no individual wave function suggests that they have no individuality. This elminativism is a radical form of structuralism in which there are relations without relata: objects are not fundamental, relations are. All there is, is structure. Given the necessity of explaining how relations can exist without relata, some have proposed more moderate structuralist perspectives which relax this constraint and generally maintain that structure and objects are both fundamental components of the world.

Valia Allori, in Chap. 10, proposes a structuralist understanding of the properties of fundamental objects. The proposal is that these objects have no other fundamental property than the one needed to specify their nature. The basic idea is that properties are in the laws, rather than in the objects, and laws may be understood as suitable structural relations between these objects. This account is then compared and contrasted with various types of structuralism, and it is argued that it is superior in various respects: it shares the main motivations of more traditional structuralist views, but it does not have the corresponding problems. In the quantum domain, this approach is compatible with both the primitive ontology approach and the wave function realism. In fact, according to wave function realism, the wave function is fundamental while everything else, including objects and their properties suitably, emerges from it, while in the primitive ontology framework all properties are in the law, including the wave function.

Another perspective on quantum mechanics with a strong structuralism component is relational mechanics. In this view, defended by **Carlo Rovelli** in Chap. 11, the wave function does not describe objects nor properties, but it is merely a calculational device. The fundamental ontology is relational, discrete, and relative. The world is a collection of facts interrelated with one another. Facts are identified as relations by interactions between other facts. There are no fundamental objects and no fundamental properties, and all our talk about them emerges from the relational ontology. The properties described by these facts are determinate only when systems interact, and they are relative, as they can always be thought of as relations between two entities.

In Chap. 12, **Francesca Vidotto** builds up on this and argues that relational mechanics is supported by the most promising proposals for fundamental physical theories, from classical mechanics to gauge theories, and from general relativity to quantum field theories. Vidotto argues that all these theories, from the past to the present one, suggest an ontology of relations, constructing an argument which seems akin to the structuralist response to the pessimistic meta induction problem: A relational ontology has been preserved through theory change from past to current theories; therefore, we are justified in believing in the existence of such relations.

Even if she does not explore this in detail, Vidotto remarks that this relational ontology helps in dissipating the worries connected with quantum nonlocality. It has been extensively argued, but it still remains controversial, that Bell has proven that nonlocality is a fact of nature. Briefly, his argument is the following: Bell, starting from the reasoning of Einstein, Podolsky, and Rosen (EPR), considered a particle source which emits pairs of spin-correlated particles in opposite directions. Experimental results of spin measurements on the two sides display perfect anti-correlations of outcomes. Since in quantum theory each particle acquires a definite spin-property only upon measurement, when the experimenter on one side finds spin up, say, along one direction, her act of measurement makes the particle on the other side acquire the opposite property, and this violates locality of interaction. To avoid nonlocality, Bell, following EPR, concluded that the spin properties have to exist before any measurement. He then proposed an inequality which would be satisfied by a such a theory but violated in quantum mechanics. Subsequent experiments established convincingly that the quantum mechanical predictions are correct, showing, in the eyes of many, that the world is nonlocal. Relational mechanics, it is proposed by Vidotto, avoids this conclusion in virtue of its relational ontology: quantum theory is nonlocal only because we think of single particles as existing independently, while this is not the case in this framework.

In Bell's proof there is also the so-called hypothesis of statistical independence, which is usually taken for granted. It states that the experimental settings do not depend on the distribution of the additional variables. One way to make this assumption false (and thus invalidate the nonlocality conclusion) is allowing for superdeterministic theories. Bell regarded them as conspiratorial, but this is the route that **Gerard t'Hooft** considers in Chap. 13. He proposes a local, determinist theory, which he dubs the "cellular automaton interpretation" of quantum theory. However, this is not an interpretation of a theory but rather a new theory itself. The quantum formalism is shown to be derivable within this theory, and it is argued that quantum mechanics is best seen as a useful tool, rather than a theory with some ontological import. Also, it is shown that this theory is local because it violates the hypothesis of statistical independence. Nonetheless, t'Hooft argues that denying this assumption is less troublesome than many have thought.

Part III: The Wave Function

The third part of this collection is devoted to the wave function: should we think of it as a possible ontology of the theory, or does it have some other role in the theoretical framework? We have already seen that some approaches, such as wave function realism, take it to represent the fundamental ontology of the world. Other perspectives instead regard, in their own ways, the wave function very differently than wave function realism.

Some quantum theories seem to go better with some of these approaches than other. For instance, the primitive ontology approach is the straightforward way of interpreting Bohmian mechanics, while wave function realism seems to be better suited for the GRW theory or Everettian mechanics. The reason is rather obvious: in the original formulation of these two theories there is no object other than the wave function, and adding another entity as the ontology of the theory, as the primitive ontologist suggests, makes them less simple. Nonetheless, wave function realism has been resisted by proponents of Everettian mechanics for some of the reasons we have mentioned but also for others. **Lev Vaidman**, who is a defender of the many-worlds theory, in Chap. 14 proposes an approach in which the wave function is objective real, like in the case of wave function realism, but not fundamental. He argues that the reason for which we have to admit that the wave function is in configuration space is to account for the nonlocal properties of the physical objects. He, however, maintains that our experience of life in three-dimensional space supervenes on the portion of the wave function defined in three dimensions. He then concludes that the many-world picture is forced upon us if we wish to avoid nonlocality in such a deterministic picture.

Another possible way of defending the many-world picture is given by **Sean Carroll**, in Chap. 15. His approach follows wave function realism in considering the wave function as fundamental but differs in regard of which type of mathematical object one should think the wave function is. Carroll defends the view that the fundamental ontology of the world is given by the wave function, which is best understood as a vector in Hilbert space evolving according to the Schrödinger equation. Carroll argues that everything else suitably emerges from this: from the laws of physics, which are determined by the Hamiltonian, to three-dimensional space itself.

At the opposite side of the spectrum of possibilities is **Jacob Barandes**. If Carroll argues that the Hibert space picture is the essence of quantum theory, in Chap. 16, Barandes instead maintains that the Hilbert formalism, including the wave function, is just a tool which has been useful. It is merely the shadow that we, prisoners in Plato’s cave, are led to believe is real. He therefore proposes what he dubs the “Platonic interpretation” of quantum theory in which systems are represented by sets of properties (as for instance, positions and momentum) whose motion is described in terms of the Gelfand-Naimark-Segal construction, which Barandes regards as equivalent to the fire in Plato’s cave allegory.

Whether one can satisfactorily recover our three-dimensional picture of the world from a fundamental ontology has been extensively discussed in the literature. In this collection, **Jean Bricmont** contributes to this debate with Chap. 17, emphasizing how the wave function is fundamentally unlikely a physical field, which prevents them to be considered suitable ontologies. Furthermore, he argues against Everettian mechanics by providing an argument of how this theory is unable to recover the statistics of the results: Bricmont argues that the measure required to give rise to the correct predictions is unjustified and thus needs to be postulated in an *ad hoc* manner.

The question which however remains open is about the nature of the wave function: if we think that the wave function does not represent physical systems, then what does it represent, if anything? As anticipated, some of the approaches discussed above share the attitude of considering the wave function as not representing anything material. However, they disagree in how exactly we are supposed to think of the wave function. A part of them, like relational mechanics, the cellular automaton interpretation, and the Platonic approach, maintain that the wave function does not represent anything in the world, neither matter, nor something else. It is merely a useful tool to formulate the theory. Others, like the proponents of the information theoretic approach, consider the wave function as representing something, but not representing something about the world. Rather, they think that it represents something about our knowledge of it. In other words, in their view, the wave function is epistemic, rather than ontic. Some others, like the primitive ontologists and the proponents of the minimalist ontology program, lean towards considering the wave function as nomological. That is, they think that the wave function is ontic, rather than epistemic. However, instead of representing matter, it represents something to describe the way in which matter moves. This idea can be spelled out in different ways: some regard the wave function as exemplifying a dispositional property of matter, some others instead think more generally in terms of the wave function akin to a potential or the Hamiltonian. Naturally, approaches may overlap: for instance, two approaches may agree in considering the wave function akin to a potential, but disagree about whether potentials are real or not (see, e.g., the primitive ontology approach and the Platonic interpretation).

There are several open questions when considering each of these perspectives. As noted already, the primitive ontology program is particularly suited to discuss Bohmian mechanics. Dispositionalism is the view according to which, within this framework, the wave function represents a dispositional property of the Bohmian particles. In Chap. 18, **William Simpson** and **John Pemberton** propose an argument against the dispositional account of the wave function. They show that the dispositions fail to determine the particles' trajectories, and they propose instead "powers with Aristotelian timing," that is, powers that persists through time. They then argue that this cosmic hylomorphism, or a teleological process, is preferrable to dispositionalism on a variety of grounds, first of all its simplicity.

More generally, **Nina Emery**, in Chap. 19, explores the consequences of thinking of the wave function as part of the law, proposing what she dubs "the governing conception" of the wave function, which she links to the idea that the wave function is whatever explains the behavior of physical objects. She observes that depending on what is meant by explanation, one is led to different types of questions: either questions about the reasons why something has happened, or reasons about why we should expect to observe a given phenomenon. Emery argues that we should understand the governing conception of the wavefunction as providing an answer to the former type of questions. She argues that this perspective puts constraints on wave function realism, in addition to ruling out pragmatic approaches.

Often pragmatic perspectives on the wave function are characterized as maintaining that the wave function does not represent anything. Rather, it is a useful tool to describe what an experimenter should expect to observe on their lab. In contrast with this characterization, in Chap. 20, **Richard Healey** discusses the various understandings of thinking of the wave function as representational. Then he defends the view that the wave function represents an objective relational property of a given physical system that describes neither its intrinsic physical properties nor anyone's epistemic state. Rather it represents the objective probabilities to certain physical events involving the system.

Part IV: Indeterminacy

Apart from question about ontology, another mysterious aspect of quantum theory is its probabilistic character. In the original theory, the wave function evolved according to two evolution equations: the deterministic Schrodinger equation when the system was unperturbed, while it would randomly collapse into one of the possible solutions during experimental circumstances. This indeterminism of the law is lost in deterministic theories such as the pilot-wave theory and the many-worlds theory, but nonetheless remains in spontaneous collapse theories.

Other questions are about the status of probabilities in deterministic theories, and whether they can be accounted for. This aspect has already been mentioned in Bricmont's paper, who was arguing against Everettian mechanics being able to reproduce the quantum probabilities.

More metaphysical questions have to do with the origin of the indeterminacy we see at the level of observation. Is it epistemic or ontological?

As we have seen, Mariani's paper put forward the possibility of having an indeterminate ontology for GRWm to account for the meaning of the indeterminate portion of the matter density field. On a more general note, in the framework of standard quantum mechanics, it is often argued that there is genuine metaphysical indeterminacy regarding the properties we associate to quantum objects. In other words, observables fail to have definite properties. In fact, in standard quantum mechanics, there is just the wave function evolving according to the Schrödinger equation, and physical objects get ascribed properties by the so-called "eigenvalue-eigenstate" link: the properties of a given system are given by the eigenvalues of a suitable self-adjoint operator. When the system is not an eigenstate of the operator corresponding to the property which is being measured, then the property is indeterminate. That is, it does not have any determinate value.

There are various accounts to characterize metaphysical indeterminacy. One is the so-called the "determinable-based" approach according to which a state of affairs is metaphysically indeterminate if it involves determinable properties without having a unique determinate. In a less precise language, the idea is roughly that the property in question could have a value, but it does not have a unique one: there could be many ("glutty") or none at all ("gappy"). Consider for instance the property of having the spin "up" or "down" in a given direction. A singlet state, which is a superposition of spin "up" and "down" in the same direction, would be an example of metaphysical indeterminacy, as the system has no definite spin-in-that-direction property. However, as advocated by the so-called "sparce view," the eigenvalue-eigenstate link is compatible with there being no such properties. People have objected to this view as being unreasonable, as it seems to imply that particles have no locations. In Chap. 21, **David Glick**, the proponent of this approach, defends it against this charge, arguing that the sparse view eliminates quantum metaphysical indeterminacy without the alleged radical implications. If one moves outside of standard quantum theory, it has been argued that there is genuine metaphysical indeterminacy in the GRW theory. The GRW theory, understood as a theory about the wave function, suffers from the so-called problem of the tails. The wave function, evolving according to a nonlinear stochastic equation, spontaneously localizes. This is the way the theory solves the measurement problem. However, after the spontaneous collapse, the wave function will not localize into a precise point but will have tails extending to infinity. That is, the wave function will not be in an eigenstate of the position operator, so the eigenvalue-eigenstate link will fail to provide any location to the system described by the wave function. To solve this problem, people have proposed to substitute the eigenvalue-eigenstate link with something else. Among the proposals, one find the so-called "vague" link, according to which a system can acquire properties in degrees, depending on how close they are from the eigenvalue of the relevant operator. This seems an instance of genuine metaphysical indeterminacy. Glick however argues that this is not the only way to interpret this situation. He instead shows that this indeterminacy can be instead seen as an instance of vagueness, and that it can be understood not as metaphysical but rather as representational, namely as having to do with the way we represent the world.

Alessandro Torza reconstructs Glick's argument that standard quantum theory does not entail metaphysical indeterminacy: since quantum theory shows no evidence of metaphysical indeterminacy, then there is not any. However, Torza argues that this implicitly assumes that there is no derivative metaphysical indeterminacy. In Chap. 22, Torza proposes an alternative way of defining the meaning of metaphysical indeterminacy: a system is indeterminate if and only if there is at least a property that the system neither has nor lacks. He first shows that his approach is suitable for quantum theory, and then he argues that there are good reasons to believe that quantum metaphysical indeterminacy is indeed derivative: the quantum formalism can be embedded in different logical spaces, namely spaces of possibilities, a (fundamental) classical logical space, and a (nonfundamental) nonclassical, or quantum, logical space; since metaphysical indeterminacy arises in the latter but not in the former, it is argued that it is derivative.

Peter Lewis, in Chap. 23, suggests thinking about metaphysical indeterminacy starting from physics itself. Lewis argues that if one starts from the usual Hilbert space formalism, one can understand indeterminacy as being similar to the one defined by Torza. Also, he notices that there are two distinctive ways of ascribing properties to systems, starting from the eigenvalue-eigenstate link: a classical and a non-classical one. For instance, a classical attribution would be to say that a non-eigenstate of z-spin lacks both z-spin up and z-spin down properties, while a nonclassical one would be to say that the system has indeterminate z-spin. While the former has been employed by the determinate-based approach, the latter is found in Torza's framework. They both lead to indeterminacy; nonetheless, Lewis remarks, they are not the only possible combinations: one in fact may ascribe properties classically in Torza's account, or non-classically in the determinable-based account, and in both cases, we would not have any indeterminacy. It is an open question which combination one should adopt, and Lewis argues it should be settled analyzing whether it is fruitful within physics.

Another approach rival to the one we have seen so far is the supervaluationist account, according to which a system is metaphysically indeterminate whenever there are multiple possibly admissible, exhaustive, and exclusive states of affairs, and it is indeterminate which of these obtains. **George Darby** and **Martin Pickup**, in Chap. 24, argue for an approach which is inspired by this. Their account suitably modifies supervaluationism in terms of situations which partially (rather than totally) describe the actual world. Their approach has been recently criticized in the literature, and in this paper, they provide a reply.

So far, we have seen examples of metaphysical indeterminacy from standard quantum mechanics and the GRW theory, both of which had to do with properties having indeterminate values. In Everettian mechanics, superpositions states are interpreted differently than in the other quantum theories: they do not ascribe indeterminate properties to the same system, rather they represent many systems each with a different determinate property. So, one may be tempted to conclude that there is no room form indeterminacy in Everettian mechanics. Nonetheless, for the theory to be successful, one needs to ensure that the various systems do not longer interact (they decohere), and therefore can be considered independent "Everettian worlds." Everettians have advocated that decoherence can ensure that the interference between these systems is negligible. However, this allows for indeterminacy to come back in: interference is not completely absent, just so small to allow, for all practical purposes, ascribing definite properties to each world. **Claudio Calosi** and **Jessica Wilson**, in Chap. 25, provide new arguments for the existence of genuinely metaphysical indeterminacy in Everettian mechanics adding to the literature on the subject. They also argue that this metaphysical indeterminacy is best accounted for by determinable-based approach when compared to their competitor, namely supervaluationism.

Al Wilson, who has first argued for the existence of genuine metaphysical indeterminacy in Everettian mechanics and from which Calosi and Wilson start their paper, in Chap. 26, instead, wishes to explore the notion of levels in the many-worlds theory in terms of various notions of fundamentality. He argues that metaphysical ground and concept fundamentality are suitable framework to understand how to combine a deterministic fundamental reality (described by the universal wave function) with an independent emergent reality (the multiple decoherent worlds).

Obviously, many other questions are left to be asked (and to be answered) about the topics the articles in this collection have discussed. Some issues are closely related to the ones covered here, and some less so. For instance, some have argued that even if a theory solves the measurement problem it still possible that it is not compatible with scientific realism. Is that correct, and if so, what are the consequences? It has been argued that the emergence of the there-dimensional world in Everettian quantum theory can be accounted in terms of grounding or concept fundamentality. How does this extend to the other approaches? Is there a notion of fundamentality which would fit best within relational or pragmatic approaches? What is the right notion of fundamentality in the primitive ontology framework? It has been suggested that the primitive ontology can be indeterminate. How would that play out? What is the relationship between relational quantum mechanics, pragmatic approaches, and structuralism? Even if these questions have been touched upon in some of the papers, a deeper discussion seems to be necessary, even if not here.

I could list many more questions and open problems. In any case, it would have been foolish to even think of trying to provide anything remotely close to a comprehensive set of contributions on topics such as these, which are currently hotly debated. In the literature, one can already find some impressive collections addressing similar issues, and there is certainly room for many more volumes to be written on them. In any case, I am confident that this collection will be of interest for physicists, philosophers of physics, and metaphysicians interested on quantum theories, their different formulations and modifications, their implications for philosophy, and the various ways to provide a naturalized ontology for them.

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