Quantum Mechanics

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Abstract

This is a brief review of the history and development of quantum theories. Starting from the experimental findings and theoretical results which marked the crisis of the classical framework, I overview the rise of axiomatic quantum mechanics through matrix and wave mechanics. I discuss conceptual problems such as the measurement problem that led scientific realists to explore other, more satisfactory, quantum theories, as well as Bell's theorem and quantum nonlocality, concluding with a short review of relativistic theories.

1-Introduction

This chapter provides a concise historical introduction to quantum theories, stressing the conceptual issues rather than the technical details. This is briefly what I will cover in this paper. In contrast with classical and relativity theories, which were developed by a single figure, respectively Newton and Einstein, which were both scientific realists (the believed that scientific theories can inform us about the nature of reality), quantum mechanics had many parents, with the most different philosophical attitudes. Also, while the proposals of classical and relativity theories were driven by desires of unification, quantum theory was born to fix some recalcitrating empirical data. I start reviewing these findings which at the beginning of the 20th century led to the development of 'old quantum theory', which matured in matrix mechanics in the 1920s. They were effective phenomenological summaries, and thus were considered by many as provisional accounts. Wave mechanics instead was proposed as deeper understanding of the phenomena but, as I review, it failed for a variety of reasons, giving free rein to the positivistic-inspired 'axiomatic quantum mechanics'. Even if in the 1930s realists proposed several arguments against this theory, like the measurement problem, axiomatic quantum theory remained unchallenged for a long time. Finally, in the 1950s, new realist quantum theories were developed, which ultimately led to the empirical discovery of quantum nonlocality in the 1980s. I conclude with how, starting from the 1940s, attempts at combining quantum theory, electromagnetism and relativity led to the development to relativistic quantum field theories, which however are now seen as effective rather than fundamental theories.

2-Old Quantum Theory

According to popular legends, quantum theory was born in 1900, when Planck introduced the concept of 'quanta' to explain the observed frequency distribution (the spectrum) of the blackbody radiation. A 'blackbody' is an idealized body which absorbs all electromagnetic radiation without reflecting it back. In contrast with the classical predictions, at a given

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temperature a blackbody emits radiation with a distribution peaked at a given frequency. Planck proposed a model to fit the data, presupposing that the energy E of the radiation could only assume discrete, or quantized, values: $E = h\nu$, where $h = 6.63 \times 10^{-24}\, m^2\, kg\, /\, s$ is Planck's constant, and ν is the frequency. The idea of quantization was later used by Albert Einstein's postulation of the photons to explain features of the photoelectric effect, for instance the disappearance of electron emission by metals exposed to radiation below a threshold frequency, and the proportionality of the energy of the emitted electrons to the incident frequency. While classically unexpected, they can be explained assuming that light is made of particles, dubbed 'light quanta' or photons. Bohr's proposal that the orbital radiuses of electrons around the nucleus are also quantized could explain the stability of the atom. Classically, accelerating charges like orbiting electrons lose energy by radiating, thereby eventually collapsing into the nucleus. Instead, while in the permitted orbits, no radiation would be emitted. The orbital quantization would also explain the discrete spectra of gases, which classically should be continuous: a line in the absorption spectrum corresponds to an electron jumping from an internal permitted orbit to an external one, absorbing a photon.

These considerations mark the beginning of 'old quantum theory', where the main goal was to propose general rules to quantize various classical quantities to amend classical physics, still thinking of matter as made of particles and of light as an electromagnetic wave. Nonetheless, experimental evidence that something deeper needed changing started accumulating: as we saw, the photoelectric effect suggested a particle-behavior for light; moreover, two-slits experiments performed with particles showed interference patterns, hinting that they also have a wave nature. However, how could that be?

De Broglie built up Einstein's proposal of the photon suggesting that to each electromagnetic wave there is an associated light particle, and that to each particle there is an associated matter wave (1923). However, his work was not well received, which effectively stopped this project for a long time (see Bacciagaluppi and Valentini 2009). This, together with the already rampant positivistic climate, contributed to the idea that a classical understanding of the quantum phenomena in terms of interacting particles and fields was impossible.

3-Matrix and Wave Mechanics

The first unified model to systematize experimental results was the so-called 'matrix mechanics', by Born, Heisenberg, and Jordan (1925). It could reproduce the quantized values as eigenvalues of suitable operators (an eigenvalue a of an operator $\hat{A}$ is the number multiplying the corresponding eigenvector $\vec{a}$ under the action of $\hat{A}$: $\hat{A}\vec{a} = a\vec{a}$). It provided a mathematically elegant formalism, in which the explanation of the phenomena refers only to what can be observed.

Following a diametrically opposed attitude, Schrödinger (1926) proposed an explanation of the quantum rules assuming that matter was wave-like in nature: for instance, some quantized values are seen as the nodes of standing waves. Schrödinger's wave mechanics took inspiration from de Broglie's idea but eliminated the particles. The wave, expressed as a function of position, is called wavefunction $\psi(r)$, and its law of temporal evolution is given by an equation. which now bears Schrödinger's name: $i\hbar\frac{\partial\psi}{\partial t} = \hat{H}\psi$ (where $\hbar = h/2\pi$, and $\hat{H}$ is the Hamiltonian $\hat{H} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial r^2} + V(r)$, describing the kinetic energy and the interaction). Particles emerge as wave-packets, superpositions of waves with different wavelengths. Shortly thereafter Schrödinger proved that matrix and wave mechanics are equivalent. Therefore, the importance of Schrödinger's work is twofold. First, since wave mechanics is formally easier to handle than matrix mechanics, it is an extremely useful mathematical tool. But more importantly (at least for Schrödinger), it provides a clear visualization of the mechanism producing a phenomenon, in contrast with matrix mechanics.

However, while they all appreciated its visualizability, Lorentz, echoed by de Broglie and Einstein, despised that the wave cannot be seen as oscillating in space and time. In fact, for an N particle system, it is a wave in $3N$ dimensional configuration space (Prizbram 1967). In response, Schrödinger proposed change density (given by the square module of the wavefunction intergraded in all but three dimensions) as the quantity with physical significance. Nonetheless, the superposition principle holds: if ψ_1 and ψ_2 are two possible descriptions of the system, then also their sum is. That means that, say, a radioactive particle could have decayed, could not have decayed, or it could be in a superposition of 'decayed' and 'undecayed'. These microscopic superpositions may be thought of corresponding to the system not having a definite state. The problem is that these superpositions propagate macroscopically, while they are never observed, as emphasized by the Schrödinger cat example: take a cat in a room with a vial of poison; if the release of the poison is triggered by a substance decay, the cat could be in a dead-and-alive superposition. This is empirically inadequate, but it is the unavoidable conclusion of these assumptions. Because he could not fix this problem, Schrödinger stopped working on quantum mechanics, while most scientist started to align with the positivistic attitude.

4-The Rise of The Orthodox View

Heisenberg initially advocated for a particle ontology. However, he soon formulated his uncertainty principle, according to which a particle position and momentum cannot simultaneously be known with absolute certainty, and concluded that even if there are particles, they do not have determinate trajectories (1927). Bohr (1927) instead endorsed wave-particle complementarity. Quantum objects are described by concepts like 'particles' and 'waves' only partially and display a dual nature. Born (1926) originally embraced Schrödinger's wave, however interpreting it as a 'probability amplitude': the square of the wavefunction $|\psi(r)|^2$ gives the probability of finding a particle in r. Be that as it may, the members of the Copenhagen school, guided by Bohr, eventually formed a compact front against Schrödinger's understanding.

While they all agreed that the quantum and the classical are fundamentally different worlds, there was disagreement about how to deal with macroscopic superposition. Bohr (1928) thought that the world was governed by two equally fundamental theories: classical mechanics for macroscopic objects with definite properties, and quantum theory for the microscopic domain, in which matter can be in superposition. Instead, Neumann (1932) eliminated macroscopic superpositions by postulating that upon measurement the wavefunction 'collapses' randomly and instantaneously into one of the superposition terms. However, what is a measurement, why and in what is it different from other physical processes? Later Wigner (1961) hypothesized that the collapse is caused by an observer's consciousness, thereby making the physical world not causally closed.

5-Incompleteness and Hidden Variables

The debate over quantum mechanics continued for years, especially between Einstein and Bohr. Einstein argued that quantum theory is incomplete: the wavefunction describes an ensemble rather than a single systems, and the complete description of a single system requires the specification of other variables, 'hidden' from quantum theory. Over the years Einstein proposed several arguments to show incompleteness. We already mentioned his first argument: quantum theory lacks an ontology because the wavefunction is not in spacetime. However, this argument was ineffective for positivists, who has no interest in ontology. We also anticipated Einstein's second argument, more compelling as it is about empirical adequacy: a Schrödinger evolving wavefunction produces unobserved macroscopic superpositions. Nonetheless, as we have seen, von Neumann's collapse eliminates the problem. This solution is unsatisfactory for a realist, as it is unclear when a process is a measurement. However, there is nothing wrong with it from an instrumentalist perspective. Finally, Einstein's third argument is that quantum theory is incomplete because it is nonlocal. This argument, first raised in 1927 and repeated many times thereafter, was most famously discussed by Einstein, Podolsky and Rosen (EPR 1935). In Bohm's reformulation (1951), one considers a pair of singlet-state particles travel in opposite directions. Neither has a definite spin property, but they acquire one upon measurement: when someone measures particle 1's spin and finds, e.g., 'up', particle 2's spin instantaneously collapses to 'down'. Einstein assumed locality, the idea that interaction travels at finite velocity, because, among other reasons, it would go against the first principle of relativity that nothing travels faster than light. Instead, the wavefunction collapse is nonlocal: it instantaneously erases all terms of the superposition but one, regardless their distance. Einstein thought that this nonlocality was evidence of incompleteness: in a complete description both particles have always had definite spin, which is revealed by the measurement, eliminating the need of nonlocality.

6-Axiomatic Quantum Theory

Nonetheless, the EPR argument did not come across as intended. Bohr thought it was about the indeterminacy principle, missing the point of nonlocality and the need of hidden variables (Bricmont 2016). In addition, von Neumann (1932) arguably proved that, against Einstein's hopes, hidden variables leads to contradiction. As we will see, this no-go theorem was proven to be flawed, but only 30 years later. In the meantime, the problem of macroscopic superposition and von Neumann's result, led most scholars to give in to instrumentalism, and to think of quantum mechanics as described by von Neumann's axioms:

1. the state of a system is given by a vector in a Hilbert space (a vector space with inner product) called 'quantum state', in Dirac notation written as $|\psi >$ ('ket'), whose position representation is called wavefunction $\psi(r)$;
2. the possible values of observable quantities (e.g. the possible energy values) are given by the eigenvalues of suitable operators (the eigenvalues E_i of the Hamiltonian $\widehat{H}$), and the distribution of experimental results is given by the so-called Born's rule (for energy, the probability of getting at time t result E_i is given by the square of the inner product between $|\psi >$ and the corresponding eigenvector $|h_i >: Prob(E_i, t) = | < h_i|\psi >_t|^2$).
3. the quantum state evolves according to the (linear and deterministic) Schrödinger equation $i\hbar \frac{\partial|\psi>}{\partial t} = \widehat{H}|\psi >$, unless a measurement is performed. In this case, the quantum state (randomly and instantaneously) collapses into one of the superposition terms of the possible eigenvectors of the operator representing the measured quantity, and the result of the measurement is the corresponding eigenvalue (for energy, the state collapses into $|h_i >$, a possible eigenvector of $\widehat{H}$, and the result is the corresponding eigenvalue E_i: $|\psi >= E_1|h_1 > + \cdots + E_N|h_N > \rightarrow E_i|h_i >$).

The Schrödinger equation is not Lorentz invariant, and thus relativistic equations were later proposed, by Kelin and Gordin (1926) and by Dirac (1928).

7-Realist Quantum Theories

As anticipated, the collapse rule is unsatisfactory from a realist perspective. Realist quantum theories are precise solutions to the problem of macroscopic superpositions, also called 'measurement problem': if the wavefunction is Schrödinger evolving and it completely describes every system, then there are superpositions of possible outcomes which means that measurements do not have results. The most promising theories satisfactory for the realist are the spontaneous collapse theory, the pilot-wave theory, and the many-worlds theory.

In the spontaneous collapse theory, also called GRW theory (from the initials of its proponents Ghirardi, Rimini and Weber 1986), the Schrödinger equation is modified such that, randomly and instantaneously, the wavefunction localizes in a small spatial region, at a rate which depends on the system's size: microscopic objects rarely collapse, while the more massive an object, the faster the superposition collapses. This theory is fundamentally indeterministic, and introduces two new constant of nature, chosen to fit the data: the localization frequency and size. These features are not ideal: arguably a deterministic dynamics would be preferrable, and constants should not be *ad hoc*. Nonetheless, it has been argued that this theory is worth pursuing for two, very different, reasons. First, since it makes different predictions than axiomatic quantum theory, it can be falsified. Second, it is arguably easier to reconcile with relativity. In fact, while GRW is nonlocal (the wavefunction collapses), and this is in contrast with the relativistic constrain that the velocity of light is the maximum velocity, yet there are relativistic invariant GRW-type theories which use only relativistic spatiotemporal structure (see, notably, Tumulka 2006).

The pilot-wave theory is Bohm's (1952) development of de Broglie's ideas, hence the name de Broglie-Bohm theory or Bohmian mechanics. In this theory there are particles, whose motion is guided by a Schrödinger evolving wavefunction. Matter is never in superposition because it made of particles, and the collapse of the wavefunction is effective: for all practical purposes, it is *as if* the motion of the particles is guided by the collapsed wavefunction. The predictions of this theory are probabilistic, even if the evolutions are deterministic because the exact positions of the particles are inaccessible (Dürr et al. 1992). Contrary to GRW, it is empirically equivalent to quantum theory, so arguably the choice of this theory over quantum theory would come from its clear ontology, not its empirical content. Arguably this theory's problem is relativity. This theory is nonlocal, not because of collapse but because the wavefunction is a function of the particles' locations at the same instant. Contrarily to GRW however, all the proposed relativistic pilot-wave theories require a preferred frame, which is not a relativistic spatiotemporal structure. Because of this, its tension with relativity seems greater than GRW's (however, see Allori 2022, 2024a).

To conclude, according to the third realist quantum theory all superpositions are real, even though we do not experience them. Originally, as proposed by Everett (1957), the wavefunction represented the perspective of a given observer. Later, De Witt and Graham (1973) reformulated it in terms of worlds. The idea is that all possible measurement results happen, each in a different region of spacetime effectively not interacting with the others, which therefore can be regarded as a different 'world'. In this sense, the 'words' are emergent (Wallace 2012). The cat is dead in one world, and alive in another. This theory has the advantage that it does not modify the practice and the formalism of quantum theory. An open problem, aside from its inflated ontology, is how to explain the probability distributions of experimental results in a deterministic theory. Several solutions have been proposed, but they are all controversial (Wallace 2024).

Has we have seen, since the 1920s realists thought there was another problem: the wavefunction, living in configuration space, cannot be straightforwardly interpreted as a physical wave (Bricmont 2016, Norsen 2017). As anticipated, Schrödinger was sensitive to that but afterwards got stuck with the macroscopic superposition problem. Be that as it may, all these theories solve the superposition problem, but not all of them solve the configuration space problem. Both GRW and Many-worlds are about a $3N$-dimensional field, and as such they are not satisfactory for realists like Schrödinger. Accordingly, it has been argued that to solve the configuration space problem all quantum theories need a spatiotemporal ontology (see Allori et al. 2008). Along these lines, Ghirardi (Benatti at al. 1996) suggested that GRW is a theory about a matter field defined in terms of the square module of the wavefunction, similarly to Schrödinger's idea. Otherwise, Bell (1987) proposed that the spatiotemporal events at which the wavefunction spontaneously localizes are the ontology of the theory (the 'flashes'). More recently, others have explored a particle ontology for GRW (Allori et al. 2014). Similarly, the many-worlds theory needs to be supplemented by a spatiotemporal ontology as well (Allori et al. 2011). Finally, one needs to interpret the pilot-wave theory as a pure particle theory, and to think of the wavefunction as part of the interaction, rather than a physical field (Allori 2013). Contrarily to this viewpoint, others instead argue that a high-dimensional ontology is not problematic, if one specifies the relevant rules to extract three-dimensionality and three-dimensional objects from it (see, most notably Albert 1996, Ney 2021). It has been argued that this difference in perspective boils down to different preferences about explanatory structure: those requiring a spatiotemporal ontology explain macroscopic objects as composed of the microscopic fundamental entities, and the macroscopic dynamics accounts for the macroscopic properties; instead, those who do not share this need are open to other types of explanation (Allori 2024b).

8-Bell's Theorem, Nonlocality and Relativity

In 1964 Bell 'saw the impossible done': he came to know of the pilot-wave theory, which is a hidden variable theory (positions complete the description given by the wavefunction), and which thus seems to blatantly contradict von Neumann's impossibility theorem. After studying it carefully, he realized that von Neumann's theorem relied on unreasonable assumptions, thus nullifying the threat to hidden variable theories. Historically, Hermann already pointed out this mistake much earlier, but she was totally ignored (Hermann 1935, Bacciagaluppi and Crull 2016). In any case, Bell, knowing of the non-locality of the pilot-wave theory, wanted to construct a local hidden variable theory instead, as Einstein would have liked. He thus went back to the EPR argument: assume locality, consider a theory in which the particles had spin properties at any time. Bell discovered that this Einsteinian completion and quantum theory make different predictions: the former obeys an inequality that the other violates. Thus, one can perform a crucial test, the most sophisticated of which was performed by Aspect and collaborators (1981), and it was found that Bell's inequality is violated. Since Bell's local hidden variable theories were constructed assuming (arguably, see later) only locality, this violation implies that this assumption is false (see, e.g. Maudlin 1994, Bricmont 2016, Norsen 2017). We already knew that axiomatic quantum theory is nonlocal, that GRW and the pilot-wave theory are nonlocal; now we also know that one cannot have a local quantum theory.

In subsequent years other, more reasonable, no-go theorems were proposed (Gleason 1957, Kochen and Specker 1967). These theorems start from Einstein's assumption that there are spin properties all along, revealed upon measurement, and they show that these properties obey mathematically impossible relations. From this, they conclude that hidden variables *à la* Einstein are impossible. However, ultimately these theorems fail. In fact, it is unreasonable to assume that the experimental results are spin properties because we cannot ignore the system-apparatus interaction. In fact, an experiment is a measurement of some property only if during the process the system to be measured is left (almost) unaffected (like a thermometer measuring the temperature). Nonetheless, in the quantum domain almost always an experiment heavily modifies the system and thus does not reveal something about the system before the process (like burning a piece of paper does not reveal any property of the paper). Thus, the spin 'properties' do obey mathematically impossible relations, but this signifies nothing, because these numbers do not represent properties: they simply report what happened to the system during the interaction. The pilot-wave theory by-passes these theorems because its hidden variable is not spin but position, which is the only one which can be reliably measured.

Notice that one can resist the nonlocality conclusion by rejecting one, very hidden, premise of Bell's inequality called the 'statistical independence': Assuming that an experimental result is independent of the experiment one decides to perform (the ball being found to be red is independent of my choice of making a color experiment rather than a shape experiment). One way of denying statistical independence is to assume that there are hidden variables which are 'superdetermined' to violate Bell's inequality (examples include t'Hooft 2016, Hossenfelder and Palmer 2020, Ciepielewski *et al.* 2020). However, statistical independence is arguably at the basis of scientific practice, because it guarantees that samples are reliable (for instance, experimenting on a sample of copper is a reliable guide to the properties of all copper), so denying this assumption has very high costs (see Baas and LeBihan 2021, Chen 2021, Allori 2024c).

9-Second Quantization and Quantum Field Theories

Extension of quantum mechanics to include electromagnetic fields began with Born, Jordan and Heisenberg (1926). Later Dirac (1927) proposed the first model of absorption and emission of radiation from matter, like when a photon is emitted by an electron jumping to a lower energy state. This is the first example of a quantum field theory, quantum electrodynamics, in which the particle number changes (e.g., from one electron to an electron and a photon). Jordan (1927) introduced the so-called creation and annihilation operators, to accommodate for such a particle change. This is sometimes dubbed 'second quantization', formulated in a space called 'Fock space'.

Perturbation theory was used to provide approximate descriptions of the matter-radiation interaction: the Hamiltonian is $H = H_0 + \lambda H_{int}$, where $H_0$is the free Hamiltonian, H_{int} the interaction, and λ is a small parameter. Correspondingly, the solution is a sum of terms, the first of which is the non-interacting system, and successive terms are smaller. During the 1930-40 it became evident that higher order perturbation theory led to infinities for quantities like the self-energy of the electron (which considers the effect of the field generated by the electron itself). Nonetheless, these ideas were successfully used in Fermi's theory of beta decay (1933), introducing the idea of the later-called weak interaction. After 1945, more general rules to deal with infinities, called 'renormalization theory', were proposed (see, e.g. Dyson 1949). Roughly the divergencies are formally shifted where they do not influence the prediction, and one distinguishes between measured quantities, as the measured mass of the electron, and the corresponding 'bare' quantities, which appear in the divergent series, to suitably tune the bare values to eliminate the infinities.

Quantum field theory supplemented by renormalization techniques led to an incredible empirical success, starting from the 50s. In 1962 Glashow, Salam, and Weinberg unified the weak and the electromagnetic theories into a single quantum field theory, using the idea of spontaneous symmetry breaking. Later it was demonstrated that theories with broken symmetries can be renormalizable, thus leading the way to the development of the Standard Model, which supposedly describes all interaction except gravity.

Historically, renormalization has always been considered some necessary evil (see e.g., Dirac 1965). Nonetheless, the development of the so-called 'renormalization group' in the 1960-70s led to the concept of effective field theory: quantum field theories are not fundamental theories; rather, they are valid only at a given scale or energy. Divergencies result from neglecting processes at lower scales, or higher energies. Thus, quantum field theories are intrinsically approximate, leaving open the question about what the fundamental theory is (Williams 2023, Swanson 2024 and references therein). From a realist viewpoint, treating these theories as effective rather than fundamental is relieving but also leaves a bitter taste. The good thing is that since they are not fundamental, quantum field theories no longer suffer from the problems discussed above: we merely require them to provide an empirically adequate description. However, they are of little interest for the realist, as they give little hint about what a relativistic, quantum fundamental theory (with a clear ontology and without an observer) would look like.

Further Readings

For the history of early quantum mechanics and the realist quantum theories, see respectively Beller (1990) and Becker (2020). For historical perspectives on the measurement problem, the problem of incompleteness, nonlocality, and Bell's theorem, see Bricmont (2016) and Norsen (2017). For the tension between quantum nonlocality and relativity, see Maudlin (1994). Instead, for quantum field theories see Williams (2023) and Swanson (2024). In addition, I recommend looking at the relevant Stanford Encyclopedia entries (e.g. philosophical issues in quantum mechanics; Copenhagen interpretation of quantum mechanics, Bohmian mechanics, Everettian quantum mechanics, many-worlds interpretation of quantum mechanics, collapse theories, quantum field theories).

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