Time and Quantum Mechanics

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Abstract

In this chapter, I discuss time in nonrelativistic quantum theories. Within an instrumentalist theory like von Neumann's axiomatic quantum mechanics, I focus on the meaning of time as an observable quantity, on the idea of time quantization, and whether the wavefunction collapse suggests that there is a preferred temporal direction. I explore this last issue within realist quantum theories as well, focusing on time reversal symmetry, and I analyze whether some theories are more hospitable for time travel than others.

1. Introduction

Debates over the nature of time, as well as space, famously began within classical physics in the Clarke-Leibniz correspondence, where Newton's disciple Clake and Leibniz respectively argued in favor of substantivalism and relationism (Perry 2024; for more on time in classical mechanics, see Farr 2024). In addition, the asymmetry of time (Fernandez 2024) is often explored by looking at the irreversibility of thermodynamics and contrasting it with the reversibility of classical physics (North 2011, Hemmo and Schenker 2024). A popular solution of this tension refers to a very special initial condition of the universe, which naturally leads one to explore cosmological theories (De Bianchi 2024). When moving to modern physics, the two theories which are often discussed when exploring the nature of time are special and general relativity (Demarest 2024, Pooley 2024), for one thing because within this framework time is no longer seen as independent of space, but they are united in a four-dimensional spatiotemporal continuum. In addition, theories of quantum gravity, which attempt to combine quantum mechanics and relativity, are frequently invoked in the debates about time most notably because time disappears from one of the fundamental equations but also because spacetime does not seem to be fundamental (Wüthrich 2024). In this paper I focus on the theory that is almost never mentioned when discussing time, namely nonrelativistic quantum mechanics. Arguably, one reason for the absence of nonrelativistic quantum mechanics from the debates about time is that there is no unique quantum theory, rather there are many 'interpretations.' I quickly overview various quantum theories in section 2. First, I briefly present axiomatic quantum theory, formalized by von Neumann (1932) and still used in physics textbooks. Then I summarize the main tenets of the most promising alternatives: the pilot-wave theory (also called de Broglie-Bohm theory, or Bohmian mechanics, Bohm 1952), the many-worlds theory (proposed originally by Everett 1957, so sometimes called Everettian mechanics), and spontaneous collapse or spontaneous localization theory (also known as GRW theory, Ghirardi, Rimini and Weber 1986). Axiomatic quantum theory is fundamentally anti-realist: it does not tell us anything about the microscopic world which we cannot directly observe, but it is only focused on accurately and precisely reproducing the experimental results. Therefore, in section 3 I start discussing what this theory can (or cannot) tell us about time as an observable property. In addition, following the idea that all quantities in quantum theories are quantized, some have

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explored the idea that time itself is discrete rather than continuous, and some others have maintained that the collapse postulate can be seen as evidence of an arrow of time. In section 4 I move to those quantum alternatives which are more compatible with scientific realism, the view that our best theories can be a reliable guide to metaphysics. I discuss how they provide not much insight about observable time and on the issue of whether time is quantized. I conclude by reviewing the arguments according to which stochastic theories like the GRW theory strongly suggest the existence of an arrow of time, and their critics, as well as whether the Everettian branching structure can bypass some logical problems with time travel.

2. Quantum Theories

2.1 Axiomatic Quantum Mechanics

In axiomatic quantum mechanics, the state of a physical system (its complete description) is given by a vector in Hilbert space (a vector space with inner product) called the state vector, usually denoted in Dirac's notation as $|\psi\rangle$, whose position representation is called the wavefunction, usually written as $\psi(r_1, \dots, r_N)$. The properties of the system are represented by self-adjoint operators. In particular, an operator $\hat{A}$ represents the observable property A of a system, whose possible values are given by the eigenvalues a of $\hat{A}$ (the eigenvalues are given by the eigenvalue equation $\hat{A}|a\rangle = a|a\rangle$, where $|a\rangle$ are the so-called eigenvectors of $\hat{A}$). One of the fundamental observables is E, the energy, represented by the operator $\widehat{H}$, the Hamiltonian, defined as $\widehat{H} = -\frac{\hbar^2}{2m}\nabla^2 + V$, where $\hbar = h/2\pi$ is the reduced Planck's constant. Usually the state vector (and thus the wavefunction) evolves in time according a deterministic equation called the Schrödinger equation: $i\hbar\frac{\partial|\psi\rangle}{\partial t} = \widehat{H}|\psi\rangle$. This evolution is deterministic (given an initial condition, the final state of the system is determined), unitary (it preserves the length of the quantum state), and linear (sums of solutions are also solutions). However,when a measurement of the property A, represented by the operator $\hat{A}$, is performed, the state vector follows von Neumann's collapse rule: it is reduced, instantaneously and indeterministically, to one of the possible eigenvectors $|a_i\rangle$ of $\hat{A}$: $|\psi\rangle \to |a_i\rangle$. Moreover, the probability of obtaining the eigenvalue a_i corresponding to $|a_i\rangle$ is: $\|\langle a_i\rangle\|^2$. This last expression is the so-called Born rule. The set of all the eigenvalues a_i of an operator $\hat{A}$ is called its spectrum.

As one can see, the notion of 'measurement' is present in the defining postulates of the theory through the collapse rule. This was deemed necessary because the Schrödinger evolution is linear, and thus, assuming its universality, it would produce unobserved macroscopic superposition. This is the so-called measurement problem: if $|\psi_1\rangle$ represents a particle being detected in region 1, and $|\psi_2\rangle$ a particle being detected in region 2, then also $|\psi_1\rangle + |\psi_2\rangle$ represents a particle in a superposition of 'being detected in region 1' and 'being detected in region 2,' which is never the case. Instead, according to the collapse rule, upon measuring where the particle is, the wavefunction collapses, indeterministically and instantaneously, either in $|\psi_1\rangle$ or in $|\psi_2\rangle$, and the particle is found to be either in region 1 or in region 2.

But what is a measurement? When does it happen? Why is it special? These questions are not important for an instrumentalist, who believes that quantum theory should merely be empirically adequate. However, they are crucial for a scientific realist, who instead wishes to use our best fundamental theories to have a description of reality: What happens to the particle in superposition state before it is detected? What causes the collapse? What makes a physical system an observer, or a physical interaction a measurement?

2.2 Realist Quantum Theories

The most promising realist quantum theories, namely theories that do not need the notion of observation or measurement at their fundamental level and therefore avoid having to deal with the questions mentioned above, include the pilot-wave theory, the spontaneous localization theory, and the many-worlds theory.

The pilot-wave theory is a theory of particles, whose trajectories evolve according to a guiding equation defined in terms of a Schrödinger-evolving wavefunction, which therefore never collapses. Since matter is made of particles, objects are never in superpositions.

The spontaneous collapse theory, as originally presented, is a theory about the wavefunction which evolves according to a stochastic and nonlinear modification of the Schrödinger equation, such that macroscopic objects quickly collapse out of superpositions.

In the many-worlds theory instead all superpositions are real, but simply suitably invisible to us: each term of the superposition describes phenomena which effectively behave independently from the phenomena described by the other terms. That is, they can be thought, for all practical purposes, as different worlds, branching out from the original superposition, hence the name 'many-worlds'.

Some have argued that a wavefunction ontology, an object which is not spatiotemporal, is unnecessarily radical. Thus, according to this perspective all quantum theories, including the GRW theory and many-worlds, should be regarded as having a spatiotemporal ontology, such as particles, a matter density field or a set of events in spacetime ('flashes'). Consequently, one should not talk about, say, GRW theory *per se* but instead one should talk about GRWp, GRWm, or GRWf (Allori *et al.* 2008). Be that as it may, these are all realist theories, and thus it makes sense to ask whether they can tell us something about the nature of time.

3. Time in Axiomatic Quantum Theory

3.1 Observable Time

As we have seen, axiomatic quantum theory does not aim at describing the world, but merely at accurately and precisely reproducing and predicting experimental results. In virtue of this, it should tell us nothing about the nature of time; whether it is fundamental or emergent, whether it is a substance or a relation, whether it is passing or not. So, there is a sense in which it is not surprising that time appears in axiomatic quantum theory as a dynamical parameter in the Schrödinger equation. Nonetheless, some of the founding fathers of the theory (most notably, von Neumann, Dirac, and Schrödinger) were bothered by the fact that, contrarily to relativity, in quantum theory time and space do not appear to be on the same footing: position is not a parameter at all in quantum theory. Rather, it is represented by a self-adjoint operator, and therefore position is an observable, as described in the previous section (observable quantities are described by self-adjoint operators, whose spectrum gives the possible experimental results). This asymmetry is important to instrumentalists, as they care about measurement results: having a time observable, which is distinct from the 'dynamical' time represented as a parameter in the equation of motion as mentioned above, and which would represent a measurable time property of a physical system, would restore the symmetry between time and space. In other words, from its instrumentalist perspective, axiomatic quantum theory does not tell us anything about the nature of space and time (however, see next section). Nonetheless, it should tell us about the observable time and the observable space, suitably represented by self-adjoint operators: where a particle can be located and at what time it will be observed at that location. Unfortunately, however, while we know that the position operator is the multiplication operator, namely $\hat{X} = x\,\cdot$, there is a theorem that shows that a self-adjoint operator corresponding to time does not exist (Pauli 1933).

This asymmetry between time and position is puzzling for at least two more reasons. First, it is unclear how to interpret the results of those experiments in which we seem to measure time, like for instance the time of arrival on a screen of a particle emitted by a source: if there is no time self-adjoint operator, what are we measuring? Second, it raises questions on the meaning of the time-energy uncertainty relation. Heisenberg, using matrix mechanics, originally arrived at the uncertainty principle, which arguably expresses the existence an intrinsic uncertainty connecting position and momentum: $\Delta x \Delta p \geq \hbar/2$. Nonetheless, similar inequalities should hold between conjugated operators, namely operators such that $[\hat{A}, \hat{B}] \equiv \hat{A}\hat{B} - \hat{B}\hat{A} = i\hbar\hat{I}$, where $\hat{I}$ is the identity operator. Dirac (1926a,b), motivated by this observation and by the symmetry concerns mentioned above, showed that there should be a time observable $\hat{T}$ conjugated to the Hamiltonian operator $\hat{H}$, and correspondingly there should be a time-energy uncertainty relation similar to the one for position and momentum: $\Delta t \Delta E \geq \hbar/2$. However, the meaning of this relation becomes mysterious in the absence of a time self-adjoint operator.

Some have argued that the asymmetry between the mathematical representation of time and space in axiomatic quantum theory is not problematic because we are still in the non-relativistic regime. Others have argued that the asymmetry is less severe than it seems, as x is the property of being in a given position which is represented by the multiplication operator, not space itself (Hilgevoord 2005, Hilgevoord and Atkinson 2011; however, see Pashby 2015). Even so, however, these observations do not touch the problem of what we are measuring during time-of arrival experiments, if not the eigenvalues of a time operator.

In any case, it should also be noted that Pauli's argument is controversial, partly because it is extremely succinct. The argument is that the existence of a time self-adjoint operator would require a generic Hamiltonian to be unphysical, as its spectrum would have to be the real line, while there are no negative energies. Nonetheless, there are some *specific* Hamiltonians for which a time operator such that $[\hat{H}, \hat{T}] = i\hbar\hat{I}$ does exist, and it has been dubbed canonical time operator (Busch 2007; see also Butterfield 2013 and reference therein).

3.2 Quantization of Time

If operators represent properties, and operators have a discrete spectrum, then properties are quantized. Position has a continuous spectrum, so position is not quantized: a quantum object can be found to be anywhere (this is not always true for bound states: for instance, in the case of the Hydrogen atom, the permissible orbits of an electron are quantized). However, the Heisenberg uncertainty principle has suggested to some that $\Delta x = \hbar/\Delta p$ is some 'minimal' distance. That is, even from its instrumentalist standpoint, axiomatic quantum theory might be taken as telling us that space itself, rather than the possible locations an object might assume, is quantized: space is discrete, and Δx represents the 'quantum of space', the minimal size of the lattice space is fundamentally constituted by. If so, and if space and time should be treated on the same footing as in relativity theory, then it seems plausible to think that time is quantized as well: there is a fundamental ‘quantum of time’ (Snyder 1947).

Regardless of whether time should be seen as a discrete or continuous variable, evolution equations on a lattice (i.e. a discrete manifold) have been proposed. Interestingly, an intrinsic discrete time interval sometimes called ‘chronon’ was also introduced to avoid divergences in classical electrodynamics (Caldirola 1956), and it was later generalized into a new quantum theory (Caldirola 1976). One of the achievements of such a theory is to explain the muon as the first excited state of the electron, instead of as another type of fundamental particle (see also Farias and Recami 2010). Be that as it may, these space-time quantization proposals arguably might be seen as precursors of what happens in quantum theories of gravity such as loop quantum gravity, in which spacetime itself has an ‘atomic’ structure, even if the fundamental entities are intertwined loops forming spin networks rather than spatiotemporal objects. Other theories of quantum gravity instead suggest that time does not exist, as the fundamental formula of canonical quantum gravity, namely the Wheeler-de Witt equation, looks like this: $\hat{H}|\psi\rangle = 0$. This formula embodies the so-called ‘problem of time’: this equation describes a ‘frozen’ state rather than its temporal evolution (see Wüthrich 2024).

3.3 Direction of Time

Even if axiomatic quantum theory is rooted in instrumentalism and it treats (dynamical) time as a parameter, the collapse rule seems to suggest something about its very nature. That is, it seems to suggest that it has a preferred direction: the fact that the wavefunction collapses, and never un-collapses, entails that axiomatic quantum theory gives different results depending on time’s direction, picking up a preferred temporal direction. This is arguably because the collapse destroys superpositions and this information about the past cannot be recovered (see, most notably, Penrose 1989).

Regardless of the reasons why there seems to be a directionality of time due to wave-function collapse, the argument has been criticized on different fronts. Informally, a theory with a privileged time direction is one which “tells a different story” in the future than in the past. Arguably, that formally means that the theory breaks the symmetry of time-reversal invariance: for any history of the world allowed by the theory, and given by the sequence of states $S_i, \ldots, S_f$ between an initial state S_i and a final state S_f, also the sequence ${S_i}^*, \ldots, {S_f}^*$ is allowed by the theory, where $S^* = \hat{T}(S)$, where $\hat{T}$ is the time-reversal operator. (Notice: this is how this operator is denoted in the literature. However, it should not be confused with the observable time introduced in the previous subsection: the two have nothing in common, as the time-reversal operator acts on solutions of the theory, while the observable time operator describes the possible properties of a time measurement.) Depending on how $\hat{T}$ (the time-reversal operator) is defined, axiomatic quantum theory changes its symmetry properties. Using a popular analogy, one can think of a possible history of the world as a movie, in which case the time reversed history would be the movie projected backwards. If so, it seems natural to think of $\hat{T}$ as an operator which simply flips time, keeping the content of the states the same. That is, $\hat{T}: t \to -t; \hat{T}(S) = S(-t)$. With this definition, axiomatic quantum mechanics turns out to violate time-reversal symmetry, not only due to the collapse rule, but also considering the Schrödinger evolution. In fact, given the state of the system S is given by ψ, the time reversed state would be $\hat{T}(\psi) = \psi(-t)$. However, a solution of the Schrödinger equation would be the complex conjugate $\psi^*(-t)$ rather than $\hat{T}(\psi) = \psi(-t)$. Wigner (1959) has therefore argued that the time-reversal operator inverts the sign of the time parameter but also suitably transforms the wavefunction into its complex conjugate. This operator is sometimes called the Wigner operator $\hat{W}: t \to -t; \hat{W}(\psi) = \psi^*(-t)$ (see Earman 2002 and Roberts 2017 for justifications of the use of $\hat{W}$ as a time -reversal operator). This makes the Schrödinger component of the wavefunction evolution time-reversal invariant. Nonetheless, the asymmetry introduced by the collapse still stands. To solve this, among other things, a time-symmetric reformulation of the axiomatic formalism has been proposed (also called ABL formalism from Aharonov, Bergmann and Lebowitz, 1964). Within ABL, axiomatic quantum theory is thus completely time-reversal invariant, so the claim that there is a preferred time direction seems ungrounded. Others have challenged this by rejecting the soundness of the use of $\hat{W}$ as a time -reversal operator, and by questioning that ABL 'dissolves' the directionality of collapse (Callender 2000; see also Lopez 2022a for a criticism).

4. Time in Realist Quantum Theories

Let's see whether we can gain some insight into the nature of time from quantum theories which can be given realist interpretations. As in axiomatic quantum theory, time is treated as a parameter in the dynamics. It should be noted that this was also the case in classical mechanics, and since this is a realist theory, the comparison is stronger than with axiomatic quantum theory. As it is known, Newtonian mechanics is compatible with different metaphysical accounts of time: just looking at the theory, one cannot definitely solve disputes over whether time is emergent or primitive; whether it is a substance or a relation; whether it has a beginning or an end, whether time passes. The situation is similar in a deterministic theory like the pilot-wave theory (even if some speculative research argues for a relational approach is to be preferred, DGZ 2020, Naranjo and Vassallo 2024). In the case of the GRW theory and Everettian mechanics, the situation is not so straightforward, as the GRW theory is not deterministic and many-worlds has a branching structure. Nonetheless, it seems sensible to suggest that, as far as these questions are concerned, they give us no distinctive insight as well.

4.1 Observable Time and Time Quantization

Time in classical mechanics can be measured, in the sense that one can measure the duration of an event or the time at which this event took place, just like one can measure its position, without the need to introduce self-adjoint operators. The same is true in the case of the pilot-wave theory (as discussed in Dürr *et al.* 2004) as well as in the spontaneous localization theory (Dürr *et al.* 2007): self-adjoint operators emerge as useful tools to describe experimental statistics so it is not necessary to assume that an observable property is necessarily associated to a self-adjoint operator. Thus, the fact that there is no time self-adjoint operator in these theories does not create the same puzzles as in axiomatic quantum theory. Notice that, since no work has been done on whether operators are fundamental or emergent in Everettian quantum mechanics, it is unclear whether the same arguments put forward within axiomatic quantum theory would still hold in this context.

Similarly, there is no reason to suppose that the truth of either the pilot-wave theory or GRW suggests that time is quantized based on the Heisenberg uncertainty principle. In fact, in both theories this principle is purely epistemic: it merely says that there is a fundamental limitation of knowledge when measuring certain quantities (Tumulka 2022). However, there are proposals of spontaneous localization theories on a lattice, arguably to facilitate their relativistic extension (Dowker and Henson 2004, Dowker and Herbauts 2004), which are compatible with thinking of space-time as quantized.

4.2 Direction of Time

What about the direction of time? The pilot-wave theory is time-reversal invariant, given that there is no fundamental collapse, and using $\widehat{W}$ as the time-reversal operator, the Schrödinger equation is time-reversal invariant. This suggests there is no compelling reason to think that the pilot-wave theory suggests that there is a preferred temporal direction. However, Allori (2019) has argued that the time reversibility of the pilot-wave theory is justified only assuming that the wavefunction does not belong to the state (that is, matter is made of particles and the wavefunction does not describe material objects), and that any other realist theory with a wavefunction ontology (such as the spontaneous localization theory and Everettian mechanics as they are conventionally interpreted) would not be time-reversal invariant (or: the claim that they are time-reversal invariant is unjustified).

In addition, Everettian mechanics by nature has a complex branching structure. Is it space-time which branches together with matter within it, or does the branching of matter happen in the same space-time? If the former, we might infer something new about the nature of time: namely that it suitably splits. However, this take on the Everettian framework has been mostly abandoned in favor of a view in which the branching worlds are seen as emergent structural properties within the quantum state, evolving in the usual space-time (Wallace 2012, 2013). Within this understanding time is seen as a parameter, as in the other theories. However, it seems relevant to notice that, even if the theory is deterministic, given its branching structure, it is not obvious how to properly define a time-reversal operator. In any case, presumably, the theory will come out time-reversal invariant only if the Wigner operator is used to represent a time-reversal transformation, since the fundamental evolution is given by the Schrödinger equation.

Since the GRW theory has a stochastic evolution, the past is naturally seen as fixed and the future as open, suggesting that time has a preferred direction (Arntzenius 1997, Callender 2000, North 2011, Esfeld and Sachse 2011). Nonetheless, some have argued that the spontaneous localization theory, seen as a theory of matter in space-time rather than a theory about the wavefunction (Allori *et al.* 2008) is time-reversal invariant (Bedingham and Maroney 2017a b; Allori 2019), while Lopez (2022b) defends the original intuition.

4.3 Topology of Time and Time Travel

According to Lewis (1976), time travel happens when the duration of the journey is different from the temporal separation between departure and arrival (see also Effingham 2024). Time in classical mechanics takes values on a one-dimensional line. Therefore, departure and arrival can only be events on such a line, and going back to the past means going back to a time which leads to the present. The possibility of time travel has been challenged on logical grounds, most famously by the grandfather paradox. The idea is that traveling through time is absurd: if I could go back in the past, I could also kill my own grandfather, preventing my own birth, and making it impossible for me to travel back in time to start with. Given that they share the same topology of time as classical mechanics, the same conclusion seems to hold also in all realist quantum theories. Except Everettian mechanics, that is. In fact, in the many-worlds theory the branches can be seen as 'parallel universes,' each producing a different timeline. These branches are something unprecedented in the history of physical theories: even if they are emergent, they are still real, and one could think of each branch as including one of our counterparts. So, within this framework, one can accommodate time travel without logical paradoxes: when I go back in time and kill my grandfather, then I create a branch in which my future-counterpart who left the present to go to the past does not exist (Deutsch and Lockwood 1994). One question, however, is whether traveling to a different branch still amounts to time travel: my counterpart who kills her grandfather never goes back to her original branch, and never returns to her friends and families. Regardless, another question is whether it is physically possible to travel in time. What this thought-experiment has shown is only that, if time travel is physically possible, then within the many-worlds theory there are no logical impediments to it. In any case, it has interestingly been argued that the possibility of time travel would break time-reversal invariance (Wallace 2012).

In passing, let me mention that the violation of time-reversal invariance is often seen as problematic because symmetries are generally considered *desiderata*. In theory formation we impose symmetries on the dynamical laws, and in theory choice we favor the theory with more symmetries. So, we favor theories which preserve symmetries arguably because, given Noether's theorem, every continuous symmetry has a corresponding conservation law, and because, more generally, a theory with more symmetries is less observer-dependent. However, lack of time-reversal symmetry arguably might be less problematic when moving to relativistic quantum field theories. In fact, the CPT theorem, which is one of the fundamental theorems of quantum field theories, states roughly that every relativistic quantum field theory has a symmetry that simultaneously reverses charge (C), reverses the orientation of space (or 'parity,' P), and reverses the direction of time (T). However, it has been experimentally observed that some interactions violate the CP transformation, so that, if the CPT theorem is true, time-reversal symmetry has to be violated.

5. Conclusions

In this paper I have reviewed how time enters in both axiomatic and realist quantum theories. In axiomatic quantum mechanics, I have introduced observable time, as distinct from the dynamical time present as a parameter in the fundamental equation of motion. I also have reviewed the arguments that the collapse of the wavefunction is evidence of an arrow of time, and critiques of these arguments. With respect to realist quantum theories, I have discussed how time has the same role in the pilot-wave theory and classical mechanics, while the stochastic evolution of the GRW theory is suggestive of a fundamental arrow of time, even if critics have argued that this is not necessarily the case. Finally, I have focused on the peculiar spatiotemporal structure of the many-worlds theory, overviewing how it has been suggested to allow for time travel.

Related Topics

Ch. 23, "Time's Arrow" by Alison Fernandes; Ch. 29, "Time and Classical Physics" by Matt Farr; Ch. 30, "Time and Special Relativity" by Heather Demarest; Ch. 31, "Time and General

Relativity" by Oliver Pooley; Ch. 32, "Time and Thermodynamics" by Orly Shenker and Meir Hemmo; Ch. 34, "Time and Quantum Gravity" by Christian Wüthrich; Ch. 35, "Time and Cosmology," by Silvia De Bianchi.

Further Readings

For introductory discussions to time and quantum theory, see Hildegvoord (2002) and Zeh (2009). For a variety of issues related to time and quantum mechanics, more technical discussions can be found in the two collections edited by Muga *et al*. (2008, 2009). A recent and interesting perspective on the arrow of time in physical theories but also quantum mechanics can be found in Roberts (2022).

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