On Quantum Mechanics and the Pilot-Wave Theory: Empirical Equivalence and Other Objections

ABSTRACT

Quantum theory and the de Broglie-Bohm pilot-wave theory are empirically equivalent. In addition to other objections to the pilot-wave theory, many physicists (and some philosophers) take this to be enough to dismiss the pilot-wave theory, as they say it adds nothing to the standard theory. In this short paper I review some objections and replies to the pilot-wave theory. In particular I respond to the empirical equivalence challenge arguing that, given their mutual relationship, there is no reason to expect the two theories to make different predictions, even if they actually might.

Keywords: quantum theory; pilot-wave theory; de Broglie-Bohm; empirical equivalence; falsificationism

1. Introduction

2025 marks the centenary of Quantum Mechanics: it all arguably started around 1900, with Planck's study of the blackbody radiation, but the first formulation of quantum theory was in 1925 in terms of Born, Heisenberg and Jordan's matrix mechanics, to continue with 1926's Schrödinger's wave mechanics. To avoid unobserved macroscopic superpositions such as 'a dead and alive cat', von Neumann's 1932 axiomatization of the theory prescribes that every physical system is described by a wavefunction, evolving in time as dictated by the equation put forward by Schrödinger, only until a measurement is performed. Then it randomly and instantaneously transforms as to provide one of the permitted values of the quantity being measured (the 'observable'). Born, Heisenberg and Jordan proposed that the observables are described by self-adjoint operators, and their experimental results are governed according to a rule, dubbed the Born rule. The empirical success of the theory thereafter has been incredible and unprecedented.

Nonetheless, the debates about what the theory tells us about the world are still raging: every day at least a new 'interpretation' of quantum mechanics is put forward. The worry is that even if quantum theory makes accurate and precise predictions, the physical meaning of its mathematical entities and of its postulates is unclear: what is matter made of? What is the wavefunction? What is a measurement? What is an 'observable'? Since these questions have no unique answer, some believe one needs to select an 'interpretation' of the quantum formalism.

One of these 'interpretations,' the pilot-wave theory, has been around since even before quantum theory was axiomatized: in fact, de Broglie proposed the basic idea of it in his doctoral dissertation in 1924, even if the theory was properly formalized and completed in 1952 by Bohm. In the pilot-wave theory matter is made of particles, not the wavefunction, a measurement is simply a particular type of physical interaction, and experimental results do not always reveal a system's property before it was experimented upon. To provide these clear answers to the questions above, the theory needs an additional equation, which describes the behavior of the particles, in addition to the one for the wavefunction. Since it was proposed as to account for the data available, this theory makes the very same predictions of quantum theory, both in practice and in principle.

This feature is often taken by many physicists (as well as some philosophers) as a defect of the pilot-wave theory, to be added to other challenges which have been raised against the theory. The argument one usually hears is that the pilot-wave theory cannot be falsified, hence, following Popper, it is pseudoscientific. Aside from that, the theory is said to be, at best, useless: what is it for, if the predictions are the same as quantum mechanics?

In this paper, I wish to respond to this and to the other objections to the pilot-wave theory. In particular, the empirical equivalence objection is often dismissed as superficial without giving a detailed answer. I am going to argue that even if the two theories are empirically equivalent, there is room for the pilot-wave theory because of their inter-theoretic relationship. In addition, I am going to emphasize how empirical equivalence holds as long as the predictions of quantum theory are precise. Since there are cases in which quantum mechanics is ambiguous, while the pilot-wave theory is not, the pilot-wave theory has a clear empirical advantage over quantum mechanics.

Here is a roadmap of the paper. In section 2, I discuss both the standard and the pilot-wave theory. In section 3, I continue explaining how predictions are derived from the pilot-wave theory, which leads to its empirical equivalence with quantum mechanics. In section 4, I review and reply to some objections to the pilot-wave theory which are not connected to experiments, which instead I discuss in section 5. To respond to these objections, in section 6, I focus on the relationship between the two theories. I argue that the pilot-wave theory is the deeper, microscopic theory from which one can derive the standard theory, and as such it makes sense that every confirmation of the standard theory is a confirmation of the pilot-wave theory. In section 7 I discuss how one might object to my characterization, and I provide a reply. Before concluding in section 9, I address some final worries in section 8.

2. Standard Quantum Theory and The Pilot-Wave Theory

Historically, the empirical data that started to become available at the beginning of the 20th century were first systematized in 1925 by Born, Heisenberg and Jordan in terms of the so-called matrix mechanics. The idea was that the statistical distribution of the experimental results about the measurement of various physical quantities could be effectively represented in terms of self-adjoint operators. In 1926 Schrödinger was able to account for the same empirical data in terms of a 'wave function' ψ which is the expansion of a more general quantum 'state vector' on the position basis. The wave function evolves in time deterministically according to a linear equation which now is known as the Schrödinger equation: $i\hbar\frac{\partial\psi}{\partial t}=\widehat{H}\psi$, where $\widehat{H}=-\frac{\hbar^2}{2m}\nabla^2+V\cdot$ is the Hamiltonian operator. In general, the possible values of an observable A represented by the operator $\hat{A}$ are given by the set of its eigenvalues α_i (defined as the numbers such that for a given vector φ_i, called the eigenvector of $\hat{A}$, one can write $\hat{A}\varphi_i=\alpha_i\varphi_i$). The probability of obtaining α_i as the result of a measurement of $\hat{A}$ is given by the Born rule: given a quantum state ψ, this probability at a given time is given by $Prob_{\alpha_i}(t)=(\psi,P_i\psi)_t$, where here P_i is the projection onto the eigenspace of $\hat{A}$ corresponding to α_i.

However, as Schrödinger himself later pointed out, the linearity of the Schrödinger equation allows for superpositions: they are natural for a wave, but they make the theory empirically inadequate. In fact, microscopic superpositions will quickly spread to the macroscopic scale, where they are never observed. This is the famous measurement problem. Assume every physical system can be in a superposition of states, such as an atom in superposition of 'having' and 'not having decayed'. Then one can construct a bigger system in which the atom is hooked up to a device which does nothing if the atom does not decay, while it breaks a vial of poison, killing a poor cat in the vicinity, if the atom decays. That means that the superposition atom (decayed-undecayed) will create a superposition vial (broken-unbroken), and a superposition cat (dead-alive), which however we never observe. The theory, if kept linear as Schrödinger prescribed, would be then falsified by everyday experience.

Hence, von Neumann postulated instead that the wavefunction undergoes a dual evolution. When it is unperturbed, if evolves deterministically according to Schrödinger's equation, while when a physical quantity is measured, the evolution changes. Following Born, Heisenberg and Jordan's assumption of 'operators as observables', upon measurement of $\hat{A}$, the wavefunction collapses, instantaneously and randomly, from the superpositions of all the possible results, to only one, say φ_{dead}. Thus, when one observes whether she is alive or dead, her wavefunction will collapse into either 'dead' or 'alive', making the theory compatible with observations.

While satisfactory for empirical purposes, this theory leaves much to be desired: why is measurement not just a physical interaction like any other? What counts as a measurement? What is the connection between measuring and observing? Does that fact that an observer is conscious make any difference? If operators represent physical properties, are they properties of what? What does the wavefunction represent?

To respond to questions like these, people started proposing what sometimes are dubbed 'interpretations' of quantum mechanics, because they are thought of as answering the last question: they provide an interpretation of the nature of the wavefunction. These 'interpretations' also usually do not refer to the notion of 'measurement', 'observer', 'observation' to provide an answer to the first set of questions. Usually, the measurement problem is formulated by stating that the following three claims are incompatible: 1) the wavefunction provides a complete description of reality, 2) it evolves according to the Schrödinger equation, 3) there are no macroscopic superpositions. Thus, one can solve this problem by denying each premise. The first strategy is followed by the pilot-wave theory (de Broglie 1923, Bohm 1952), the second by the spontaneous collapse theories (Ghirardi Rimini Weber 1986), while the third by many-worlds theories (Everett 1957).

In this paper I am going to focus only on the pilot-wave theory. In this theory, the complete description of a physical system (its state S) is not given by the wavefunction alone, but it needs to be completed by the position of all the particles composing a given system: $S = (X_1, \dots, X_N, \psi)$. The particles evolve according to a guiding equation, which for a particle k with mass m_k, one can write as: $\frac{dX_k}{dt} = v_k\,(x,t) = \frac{\hbar}{m_k}\Im m\frac{\nabla_k\psi}{\psi}$ (Bohm 1952; Dürr Goldstein Zanghì 1992). The wavefunction of the universe always evolves according to Schrödinger's equation, and this is the case under certain circumstances for the wavefunction of a subsystem.[1] In this theory everything is made of particles, and it has been argued that the wavefunction is better seen as an ingredient of the law of evolution of the particles. That is, the wavefunction has to do with the interaction between particles, like a potential, rather than representing matter (Allori 2021 and references therein).

The theory solves the measurement problem because the cat, made of particles, is never in superposition. The wavefunction will be, but due to interaction with the environment, the components of the superposition

[1] We can always divide the wavefunction as depending on the coordinate of a system composed of x particles and its environment composed of y particles, so that $\psi = \psi(x,y)$. If wavefunction decouples, $\psi = \psi_1(x)\psi_2(y)$, one can show that then the wavefunction of the subsystem $\psi_1(x)$ follows Schrödinger's equation (see Dürr Goldstein Zanghì 1992).

('alive' and 'dead') cannot longer interact. Thus, one can effectively think of the wavefunction as collapsed into the superposition component in which the particles of the cat lie and practically forget about the other.

3. Empirical Predictions and Empirical Equivalence

The pilot-wave theory is empirically equivalent to the standard theory because both prescribe that the empirical data is distributed according to the Born rule. Let us see where the Born rule comes from in the pilot-wave theory.

One can rewrite the guidance equation as: $v_k(x,t) = \frac{j_k(x,t)}{\rho(x,t)}$, where $j_k(x,t) = \frac{-i\hbar}{2m_k}(\psi^*\nabla_k\psi - \psi\nabla_k\psi^*)$ is the wavefunction analog of the Poynting vector in electromagnetism (even if in the literature is almost always called 'probability current') and $\rho(x,t) = |\psi|^2$is the wavefunction analog of the field energy density in electromagnetism (again, even if it is usually called 'probability density'; see Norsen 2018). From the Schrödinger equation, and the definitions of j and ϱ, it follows that they are connected *via* a continuity equation: $\frac{\partial\varrho}{\partial t} + \nabla\cdot j = 0$ or $\frac{\partial\varrho}{\partial t} + \nabla\cdot(\varrho v) = 0$. On another hand, as a consequence of the definition of the velocity field, it follows that if the configuration is random and distributed at some initial time according to the distribution P, then P will evolve according to the same equation: if P and ρ are identical at one time, they will remain identical at later times. Now assume exactly that: at time $t = 0$ the distribution of the particle configuration is given by $P = P_B = |\psi|^2$. This is the so-called quantum equilibrium hypothesis. From what we have just seen, then it will be distributed like that at any time: this property is known as equivariance.

It remains to justify why the quantum equilibrium hypothesis is true. For now, assume it as a postulate; we will come back to it by the end of the session.

Now consider the measure of some 'observable' A. In the pilot-wave theory the configuration of the system and the measurement apparatus, can be written as $q = \{x.y\}$ (x for the system, y for the apparatus). The wavefunction before the measurement will be in general a superposition: $\psi(x,y,0) = \sum_i \alpha_i\varphi_i(x,0)\,\phi_0(y,0)$; where φ_i is the eigenvector of $\hat{A}$ corresponding to the eigenvalue α_i, and ϕ_0 is the apparatus in its ready state. This will evolve into $\psi(x,y,t) = \sum_i \alpha_i\varphi_i(x,t)\,\phi_i(y,t)$, which is an entangled superpositions. However, in the pilot-wave theory the actual measurement outcome will be displayed in the actual configuration of the apparatus Y. If the actual configuration of the composite system initially is $Q(0) = \{X(0), Y(0)\}$, and if the quantum equilibrium hypothesis is true, namely the initial distribution is $|\psi(x,y,0)|^2$, then by equivariance the distribution of configurations is also $|\psi(x,y,T)|^2$ at time T after the measurement process. Each term of the superposition $\alpha_i\varphi_i(x)\phi_0(y)$ will evolve into $\alpha_i\varphi_i(x,t)\phi_i(y,t)$, where each ϕ_i is narrowly peaked around configurations pointing at α_i, and almost zero elsewhere (otherwise it would not be a measurement). Thus, the various ϕ_i have non-overlapping support, so that cross terms will vanish, and the probability of obtaining result α_i is given by the Born's rule.

To summarize, then, there are two steps:

1) Assuming the quantum equilibrium hypothesis, namely that initial particle configurations are $|\psi|^2$ distributed, it follows that this distribution always holds.
2) Performing a measurement of an observable reduces to making a position measurement, so from (1) it follows that empirical distributions for subsystems follow the Born rule.

The first conclusion just follows mathematically from the theory's equations and the quantum equilibrium hypothesis. As anticipated, proponents of the theory wish to argue that one can actually derive the validity of the quantum equilibrium hypothesis. One proposal, dubbed dynamical relaxation program (Bohm 1952, Valentini 1991), aims to show that even if the configuration is not in equilibrium, eventually it will reach it

(Valentini Westman 2005). This would explain why we see quantum equilibrium now. An alternative approach argues that there is nothing to explain: while it may not be true that initially all particle configurations are $|\psi|^2$ distributed, most will be. On other words, the typical initial configuration, where the measure of typicality is given by $|\psi|^2$, is $|\psi|^2$ distributed (Dürr Goldstein Zanghì 1992).[2]

The second point also follows directly from the equations. Notice that it is particularly interesting because it shows that operators do not have the same role in this theory and in quantum mechanics. In standard quantum theory every measurable property is supposed to be associated to an operator: position is associated to the multiplication operator $\hat{x} = x\,\cdot$, momentum to the differential operator$\hat{p} = -i\hbar\nabla$, energy to the Hamiltonian $\widehat{H} = -\frac{\hbar^2}{2m}\nabla^2 + V$, and so on. And the measurement of some operator, say $\widehat{H}$, reveals the property the system had before the measurement, which is mathematically one of the eigenvalues of $\widehat{H}$, say energy E_i. In the pilot-wave theory, instead, the measurement results describe where the configuration of the apparatus ends up being after the interaction with the system. It turns out that the statistics are effectively described in terms of the operator $\widehat{H}$ and its spectrum, as we have seen. Nonetheless, operators do not reveal any pre-existing property of the system. This is what no-hidden-variables theorems (no-go theorems) actually show: assuming that eigenvalues reveal properties of the system before the measurement leads to contradictory relations (Bell 1966). Notice that these theorems were taken to show that one cannot complete quantum mechanics with some hidden variables (von Neumann 1932, Gleason 1957, Kochen Specker 1967). This is true if the hidden variables are the properties associated with the operators. Nonetheless, in the pilot-wave theory the only property the system has is its spatial location, in perfect agreement with the conclusions of the no-go theorems (for a nice discussion, see Lazarovici *et al.* 2018).

4. General Objections to the Pilot-Wave Theories and Replies

Many objections have been raised against the pilot-wave theory. Some of them have not much to do with predictions. In this section I am going to review them quickly to focus on the issues of empirical equivalence in the next section.

First, some of them are simple misunderstandings. Most notably, the no-go theorems were taken to show that any attempt to complete quantum mechanics deterministically leads to contradiction. However, as quickly reviewed in the previous section, this is not the case. The result holds if one adds a hidden variable for any observable. This is not what the pilot-wave theory does: it merely adds positions. What these theorems show is that indeed, one cannot do better than what the pilot-wave theory does, and that there are no other intrinsic properties.

Some others have complained that the guidance equation is mathematically inelegant or unjustified. Setting aside issues regarding the importance of elegance and simplicity in this context, there seems to be an objective sense in which the guiding equation is simple: given that there is a current, the velocity is proportional to the

[2] They argue that the $|\psi|^2$ measure is natural because it is time-translation invariant. Bricmont (2001, 2020), Valentini (2001, 2020) argue that this is circular: you get the Born rule for subsystems only because you stipulate is for the wavefunction of the universe. We do not mean to resolve this issue here. I recommend reading Norsen (2018), who argues that both accounts can inform each other. He shows that the dynamical relaxation program relies on the notion of typicality too (it's not the case that all configurations will reach equilibrium) but ultimately one can also show that most configurations will be $|\psi|^2$ distributed for reasonably smooth typicality measures, not just $|\psi|^2$(so even if the $\lceil\psi\rceil^2$is not justified as natural, it does not really matter).

current. Moreover, it has been argued that it follows from requiring symmetry properties such as Galilei invariance, rotation invariance, etc. (Dürr Goldstein Zanghì 1992).

On a different note, some have argued that the particles in the theory have no essential roles in solving the measurement problem and that one would still have to accept a many-worlds picture (Brown Wallace 2005). The idea is that both in the pilot-wave theory and in the many-worlds theory one must take the wavefunction as physically real. In the pilot-wave theory, this is the case also for the part of the wavefunction which contains no particle. Thus, in both theories there are parallel worlds, so what is the use of the particles? As a reply, one would point out that there is an ambiguity of what it means that the wavefunction is 'physically real:' while in the many-worlds theory, since there is nothing else, the wavefunction is physically real in the sense that it represents matter, this is not the case for the pilot-wave theory. While the wavefunction is an objective feature of the world (it is 'physically real' in this sense), many pilot-wave theorists would deny that matter is represented by the wavefunction (it is not 'material'). Rather, as anticipated, the wavefunction is better understood as more like describing the interaction between matter, similarly to what a potential, or the Hamiltonian, do: it is an ingredient which helps determining how matter moves, not as something which describes what matter is (which is composed of particles). If so, then superpositions in the wavefunction do not entail superposition of matter (Allori 2021).

Another worry is the theory's asymmetry: the wavefunction acts upon the particles guiding their trajectories while the contrary does not happen. Again, this objection has some strength if one thinks of the wavefunction as representing matter, like electromagnetic fields represent light: charged particles generate electromagnetic fields, and the electromagnetic fields affect the particles; so, if matter is made of particles and the wavefunction, why don't they influence each other in the same way? However, we have just seen the wavefunction is not to be seen in this way. If we take the wavefunction as similar to the Hamiltonian, the objection evaporates: why should one expect the Hamiltonian to be 'acted upon' by the particles?

Another common objection is that the theory is incompatible with relativity. The problem with relativity has to do with the fact that relativity is local while the pilot-wave theory is not. According to relativity there is a maximum velocity at which anything can travel, namely the velocity of light. If one thinks that interaction is not instantaneous, namely that it takes time for a particle in some location to 'feel' another particle, then interaction can travel at most at the velocity of light. This is the case in electromagnetism, where the electromagnetic fields, the mediators of the electromagnetic interaction, propagate at the velocity of light. Instead, in the pilot-wave theory, since the wavefunction contains the particles configurations at the same instant, we have instantaneous action at arbitrary distance, and this is against relativity. This is indeed a problem, but not only for the pilot-wave theory: all theories reproducing the quantum predictions face the same difficulty. This is evident in the case of quantum theory with collapse, as the collapse is manifestly nonlocal (Einstein put forward this argument first in 1909 at a meeting in Salzburg; see Bacciagaluppi Valentini 2009, p. 198). Regardless, Bell (1964) showed that local theories would make different predictions than quantum theory, and Aspect (1981) later falsified them. For more discussion on relativity and the pilot-wave theory, see Allori (2025) and references therein. A common way for quantum theory to extend it to relativity is to ignore the problem of nonlocality and make the theory Lorentz invariant. This is what quantum field theories do, predicting, among other things, the creation and annihilation of particles. Now the worry is that an ontology of particles is inadequate in this context and that it cannot be extended to cover the results quantum field theories. Nonetheless, this is not the case: several pilot-wave theories which are relativistic in this way can be constructed (both as theories of fields, and as theories of particles), and they are all able to account to the same phenomena as quantum field theories. For more on this, see Tumulka (2018) and references therein.

5. Objections about Empirical Equivalence and a First Reply

Other objections directly follow from the empirical equivalence with standard quantum theory. The arguments look like this: since the two theories are empirically equivalent, the pilot-wave theory is either (A) unscientific, because it cannot be falsified, or (B) useless, because the predictions are the same (see e.g., Heisenberg 1955, Leggett 2002).

Some scholars, notably Bricmont (p.c.), would immediately block this type of argument as follows: in order to have empirical equivalence you need to have two theories, while standard quantum mechanics is not even a theory. It is merely a description of measuring devices. For the sake of the argument, instead, I am going to grant standard quantum mechanics the status of physical theory to see where the objections lead us.

Let us elaborating on (A), namely that the pilot-wave theory is not falsifiable. According to Popper's falsificationism, a theory is scientific if it can make predictions which can be proven false. This is true for both quantum mechanics and the pilot-wave theory: they both can make predictions that can be proven false. So, the charge cannot really be the one described. Perhaps what they have in mind is that there cannot be a crucial experiment that can be used to rule out one of the two theories. That is, if theory T_1 predicts that in a given circumstance one will observe O_1, while T_2 predicts that in the same circumstances one will see $\sim O_1$, then perform the experiment; if one gets O_1, then T_2 is ruled out; otherwise T_1 is. In the case at hand, there is no experiment in which the pilot-wave theory and standard quantum theory disagree, so there is in principle no possibility of setting up a crucial experiment. From this, it is concluded that the pilot-wave theory is at fault. But why is that? One, very shallow, response is that because it came later. As we saw, this is not even true: de Broglie's thesis, in which the theory was proposed, predates the development of matrix mechanics. In any case, why should the time of discovery of a theory have something to say about its value? The reason why one cannot set up a crucial experiment is simply that the theories were constructed to account for the same data, and priority of discovery does not play any role in determining which theory is more scientifically plausible. Even forgetting about this, falsificationism has many problems, and it has been long abandoned as the sole criterion for theory selection (see, e.g. Duhem 1955).

The idea of (B) is that if you have two theories T_1 and T_2, which are formally different but are identical in all their empirical content, then T_1 and T_2 are, for all that matters, the same theory. So, it is going to be useless to distinguish between the two; simply use the formalism which is more convenient to solve the problem at hand to extract the predictions. This attitude is anti-realist: T_1 and T_2 do not give us a picture of the world. Rather, they are useful tools to systematize the empirical data and to make new predictions. The argument is that T_1 and T_2 are underdetermined by the data, so one cannot choose either as 'the true' theory. Realists will not accept this, and they will argue that there are other virtues beyond empirical adequacy, such as simplicity and explanatory power, which can be used to break the underdetermination. Therefore, a common reply to this charge is claiming that even if the predictions are the same, the pilot-wave theory explains the phenomena better. While in standard quantum mechanics we are left with many unanswered questions about the nature of matter, the nature of measurements, the nature of the wavefunction, in the pilot-wave theory the answers are clear: matter is made of particle, a measurement is an interaction between two system which leaves the status of the system fundamentally unaffected, and the wavefunction describes the particle motion. We understand any phenomenon in these simple terms, while this is not the case of quantum theory. If we regard explanatory power as a super-empirical virtue which has more than just a pragmatic value, then the pilot-wave theory is more likely to be true than the standard theory.

Nonetheless, critics have pointed out several problems with this reply. First, what constitutes a scientific explanation is controversial (for more on this, see Woodward Ross 2021 and references therein): has one explained a phenomenon if one can derive it from a scientific law (as in the deductive nomological model) or should one simply identify the cause, or the causal mechanism of that phenomenon? Moreover, it is difficult to effectively argue that explanatory power is not just an epistemic virtue (see, notably, van Fraassen 1980): why should a theory which explains better be more likely to be true? Therefore, many are not convinced by this reply. In the next section I am going to propose another reply.

6. The Pilot-Wave Theory as Constructive Quantum Mechanics

Usually, two empirically equivalent theories may happen to make the same empirical predictions, but they are independent in many important respects. Take Copernican and Ptolemaic astronomy: they were empirically equivalent for a long period of time, but they depicted mutually incompatible realities, driven by unrelated ideas and values.

One talks about empirical equivalence also when there is inter-theoretic reduction. That is, one has reduction of one theory T_2 to another theory T_1, when T_2 which is more general than T_1, and T_2, and T_1 make the same prediction under the conditions under which one has the reduction of one theory to the other. For instance, Newtonian gravity is more general than Kepler's laws, which however can be derived under certain assumptions (for instance, that one body is much more massive than the other). Thus, Newtonian gravity reduces to Kepler's laws under these circumstances. That means that both Kepler and Newton, in the domain of validity of both, will make the same predictions: they will both say, for instance, that the Sun will be one of the foci of the elliptical orbits of the planets around the Sun. Consider this other example. In Newtonian mechanics all bodies are made of particles, including macroscopic bodies. So, in principle, if we could know all the initial conditions for all particles and if we could solve Newtons equations for all of them, we could predict the behavior of macroscopic systems. However, this is unfeasible: we have neither all the information nor the capabilities to carry out these calculations. Thus, one uses statistical reasonings. This is what statistical mechanics does: it predicts the behavior of macroscopic bodies such as gases in terms of statistical analysis of the particles. Boltzmann was able to show that one can find the laws of thermodynamics starting from statistical mechanics under suitable approximations. Consequently, these two theories make the same predictions when they are both valid, namely in the macroscopic domain.

Contrary to what one might think, I believe that standard quantum mechanics and the pilot-wave theory are *not* independent theories, but they stand in a reduced-reducing relationship, like thermodynamics and statistical mechanics. In other words, quantum theory, being essentially a theory reproducing measurement outcomes distributions, is a macroscopic theory which is (at best) like thermodynamics. Instead, the pilot-wave theory provides a deeper description of what correspondingly is going on at the microscopic level. There is a distinction between expecting a macroscopic phenomenon to happen, and accounting for why it happens in terms of the motion at the microscopic level. Thermodynamics and quantum theory tell us what to expect macroscopically: for instance, we should expect freely evolving systems to increase their entropy, and we should expect measurement outcomes to be distributed according to the Born rule. Correspondingly, statistical mechanics and the pilot-wave theory 'lift Maya's veil' and accounts for the microscopic mechanism which gives rise to these predictions. Just as statistical mechanics describes a gas as a collection of point particles, so the pilot-wave theory describes a quantum system as a collection of particles. Statistical mechanics can account for the laws of thermodynamics in terms of the motion of particles, and the pilot-wave theory can account for the quantum rules in terms of the motion of particles, as we have seen in the previous sections. For instance, as shown by Boltzmann in statistical mechanics, the second law of thermodynamics is statistically derived from the microscopic motion of particles, if we suitably understand entropy as proportional to size in configuration space. Similarly in the pilot-wave theory, understanding experiments as physical interactions, one can derive the Born rule only assuming there are particles moving according to the guidance equation. To put it another way, the pilot-wave theory provides a deeper understanding of the quantum phenomena, just like statistical mechanics provides a deeper understating of the thermodynamic phenomena.

Using Einstein's terminology (1919), statistical mechanics and the pilot-wave theory are the constructive theories grounding, respectively, thermodynamics and quantum mechanics, which are instead principle theories. A theory is said to be constructive when macroscopic phenomena are accounted for in terms of their microscopic constituents' dynamics. For instance, for a gas energy is conserved is explained in statistical mechanics in terms of the energy of the particles of the gas. Instead, a principle theory is one in which there are constraints that exclude various behaviors as unphysical. That is, the principle 'energy is conserved' makes it to be expected that one will never see a phenomenon in which energy is not conserved. Similar case is the one of entropy discussed above. That is, constructive theories can explain the principles used in principle theories, thus constructive theories are in this sense 'deeper.' The main principle of quantum theory is the Born rule: we expect to find empirical outcomes distributed accordingly. Since the pilot-wave theory can derive, for example, the Born rule and the various operators from its equations of motion, it provides a constructive understanding of the quantum phenomena.

For Einstein, we accept principle theories as provisional, only when there are no constructive alternatives, and that physics should aim at finding constructive theory. In this vein, then quantum mechanics being a principle theory is provisional, and the pilot-wave theory, being its constructive counterpart, is exactly what scientific methodology dictates.

If that is the mutual relation between quantum mechanics and the pilot-wave theory, then one should expect empirical equivalence in their common domain. Since it makes no sense to say that statistical mechanics is useless because it makes the same predictions of thermodynamics, it makes no sense to say that the pilot-wave theory is useless because it makes the same predictions of quantum mechanics. The appropriate thing to say is that every test for standard quantum theory is also a test for the pilot-wave theory, just like any test for thermodynamics is a test for statistical mechanics.

7. Follow-up Objections and Replies

One could resist this reading and reply that the analogy with statistical mechanics does not work. In fact, statistical mechanics (the deeper theory) makes more predictions than thermodynamics. These predictions are true for systems which are not big enough to be analyzed statistically. For instance, in statistical mechanics the second law of thermodynamics that entropy always does not decrease, is true only statistically: so, entropy is allowed to decrease in special circumstances.[3] In other words, if T_2 is deeper that T_1, then there would be

[3] A vivid case of entropy decreasing is given in heavy nuclei interactions and subsequent decay: the particles of the heavy nuclei colliding thermalize (that is, they reach thermal equilibrium, thus maximum entropy), but given the relatively small size of the nuclei, it is common for them to escape equilibrium and thus to decrease their entropy (see e.g. Gadioli Hodgson 1995).

circumstances in which the former predicts something which the latter does not. This is the case for statistical mechanics and thermodynamics, but it is not the case for the pilot-wave theory in which the predictions are supposed to be the same at all scales. Let me provide the following replies.

For one thing, one could question the empirical equivalence of the pilot-wave theory and standard quantum mechanics exploring the possibility of non-equilibrium. That is, if the universe was once in quantum non-equilibrium, then it would have displayed predictions which differ from quantum theory, as the empirical results would not be $|\psi|^2$ distributed (Valentini 1991, 2002). This would not be the case if there was never a period of non-equilibrium.

Also, one can notice that the empirical predictions of the pilot-wave theory and quantum mechanics are the same only insofar as there is an operator corresponding to (what we think is) the property being measured. Instead, this is not always the case: as it is well-known, there is a theorem stating that there cannot be a time operator (Pauli 1933).[4] Thus, experiments which involve a time measurement, such as tunneling times, escape times, and time of arrival do not have a straightforward characterization in terms of a suitable self-adjoint operator. For instance, physicists disagree about how to compute the probability that the particle's time-of-arrival on the detector, which is about 'when' - rather than 'where'- the detector clicks (Muga Leavens 2000). In the pilot-wave theory instead it is possible to define unambiguously the arrival time of a particle at any point based on the precise calculation of the trajectory passing through that point (Leavens 1998).

It has been recently argued (Das Dürr 2019, Das Dürr Noth 2019) that it is possible to set up a time-of-arrival experiment in which the predictions of the pilot-wave theory and the ones of quantum theory disagree. This has been resisted by some (Goldstein Tumulka Zanghì 2024a,b) on the basis that the calculations did not sufficiently account for the system-detection interaction. Better models describing this interaction seem to be needed to settle the dispute (see Das 2025, Drezet 2025).[5] In any case, it seems that the objection against the analogy between statistical mechanics and the pilot-wave theory loses its grip: both theories make broader predictions than respectively thermodynamic and quantum theory.

8. Anything Else?

To conclude, let me address one last set of objections aimed at putting into doubt the physical plausibility of the particle trajectories.

First, it has been argued that the pilot-wave trajectories are physically not meaningful (Englert *et al.* 1992). Consider a test particle in a two-slit interferometer, which interacts with another quantum system plays the role of a 'which-way' detector, indicating through which slit the test particle went. It is argued that in some cases the detector seems to indicate that the particle passed through one slit, while the reconstructed trajectory goes through the other. Given this surprising behavior, the suggestion is that trajectories of the pilot-wave theory are 'surrealistic' rather than realistic.

Various replies have been put forward. Some have proposed a simpler version of this experiment which shows how the effect is due to the nonlocality of the theory (Dewdney Hardy Squires 1993). Some others have maintained that surrealistic trajectories occur only from an incorrect use of the formalism of the theory (Hiley 2006). In particular, it has been observed that a proper description of the apparatus radically changes the situation (Tastevin Laloë 2018).

[4] Nonetheless, this is controversial; see Muga *et al.* (2008).

[5] Actually, Goldstein Tumulka and Zanghì argue that it is a theorem that the two theories will always make the same predictions, so they dismiss these arguments.

This discussion makes a nice transition to another connected worry, namely that there is no evidence of the presence of particles, because all empirical data can be recovered by the wavefunction in terms of the Born rule.

Strictly speaking, it is false that there is no evidence for particles: we see localized scintillations and we see tracks in detectors, so we see particles. Every evidence that, say, an electron was a particle, is evidence for the pilot-wave theory. Presumably the worry is about situations like the double-slit experiments whose results seem incompatible with particle trajectories. Nonetheless, we think they are incompatible because we expect classical trajectories, but the guidance equation predicts highly nonclassical ones. Perhaps the worry is that we cannot measure trajectories because of the uncertainty principle: every time we try to measure a particle's position, we lose information about its momentum, and thus we cannot empirically reconstruct their trajectories. This is, however, something about the possibility of extracting from the system some of its properties through measurement, and the fact that the pilot-wave theory predicts that we cannot know everything should not be taken as a problem for the theory. In any case, one can reconstruct the particle trajectories using the so-called *weak measurements* of velocities (Aharonov *et al.* 1988, Weisman 2007). The idea is that one first measures the position of the particle without disturbing the wave unction very much. Since that measurement is weak, one can do an ordinary ('strong') measurement of position a little later to obtain the precise location. By repeating that operation many times, one can produce a statistical distribution of positions, and by taking its average, obtain a position at the first location. With two consecutive positions and a time interval between them, one can compute a velocity to associate with the first position. This leads the true velocities in the pilot-wave theory (Dürr Goldstein Zanghì 2009). By repeating this in different places, one obtains a velocity field from which one can recover the trajectories as tangents. The agreement between the predicted (Philippidis *et al.* 1979) and the experimental trajectories (Kocis *et al.* 2011) is remarkable, and highly suggestive that the trajectories should be taken seriously.

9. Conclusions

In this paper I have discussed some prominent objections to the pilot-wave theory, focusing on the worry that the theory is useless because it makes the same predictions as quantum mechanics. This objection is almost never discussed in details in the literature presumably because it is taken to be connected to a naïve positivistic attitude which is not worth addressing. I disagree with this choice: by analyzing in detail what empirical equivalence amounts to, and specifically where the predictions of the two theories are coming from, one can gain a better understanding of both theories, and of their relationship. I have argued that the pilot-wave theory is the microscopic theory underlying quantum mechanics, similarly to the case of statistical mechanics and thermodynamics. If so, it does not seem sensible to ask the pilot-wave theory to make different predictions than quantum mechanics: if the idea is that the pilot-wave theory is the more general theory, then it is supposed to make the same predictions as quantum mechanics in the known cases. It usually complained that the pilot-wave theory should make more predictions, and I have discussed how it may be the case, and where new research is needed.

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