From Einstein's Statistical Interpretation to the de Broglie–Bohm Pilot-Wave Theory
Reductionism, Locality, and the Incompleteness of Quantum Mechanics

Valia Allori[1]

Abstract

This paper reconstructs Einstein's case for the incompleteness of quantum mechanics and argues that its failure was due not to reductionism, but to the assumption of locality. I show that while Einstein's attempt to locally complete quantum mechanics with non-contextual hidden variables is impossible, the core insight behind his statistical interpretation—that the wavefunction alone is incomplete—can be retained once locality is abandoned. I argue that the de Broglie–Bohm pilot-wave theory provides a non-naïve statistical interpretation that completes quantum mechanics with a spatiotemporal ontology, accommodates contextuality and fundamental nonlocality, and remains effectively local at the macroscopic scale, thereby realizing Einstein's original explanatory aims.

Keywords: Einstein; statistical interpretation; incompleteness of quantum mechanics; reductionism; locality and nonlocality; hidden variables; contextuality; Bell–Kochen–Specker theorem; de Broglie–Bohm pilot-wave theory

1.Introduction

2025 has been nominated the International Year of Quantum Science and Technology, marking one hundred years since the development of quantum mechanics—one of the most successful yet controversial physical theories ever proposed. Quantum theory is often described as revolutionary, though it is frequently unclear in exactly what sense. It has been claimed that it forces us to abandon deeply entrenched commitments such as determinism, the causal closure of the physical world, the possibility of objective description, and, in extreme cases, even the validity of classical logic. In this spirit, quantum mechanics is commonly presented as a genuine paradigm shift, requiring an entirely novel way of understanding physical phenomena.

Whatever historical and sociological factors led to the prominence of these claims, we now know that none of these radical moves is strictly necessary. One may choose to embrace indeterministic laws, allow consciousness to enter the physical description, reject realism, or revise logic—but there also exist quantum theories in which none of these steps is required.

[1] Philosophy, University of Bergamo, valiaallori@fastmail.com. www.valiaallori.com

Everettian many-worlds, the spontaneous localization theory of Ghirardi, Rimini, and Weber, and the de Broglie–Bohm pilot-wave theory are prominent examples.

Although these alternatives were developed mostly after the 1950s, the historical record shows that from the very beginning—while Bohr and the Copenhagen school were advocating a radical reinterpretation of physical concepts—not all physicists were prepared to embrace a conceptual revolution. Figures such as Lorentz, de Broglie, Einstein, and Schrödinger instead sought to determine whether quantum phenomena could be understood using what might be called the *old tools* of physics. More precisely, they aimed to construct an empirically adequate theory employing the classical explanatory schema: a visualizable account of microscopic entities, evolving in space and time, composing macroscopic objects, and interacting locally. This commitment to classical reductionism—matter in spacetime, composing objects and interacting through local dynamics—forms the background against which Einstein's critique of quantum mechanics must be understood.

Motivated by this reductionist ideal, Einstein famously argued that quantum mechanics is incomplete. His response was not to reject the theory's empirical success, but to reinterpret the wavefunction statistically: as describing ensembles of similarly prepared systems rather than the physical state of an individual system. Einstein's statistical interpretation was meant to preserve a spatiotemporal ontology, render probabilities epistemic, and sustain a constructive microphysical explanation of quantum phenomena.

The central claim of this paper is that Einstein's project did not fail because of its reductionist ambitions, but because of its commitment to locality. Once locality is abandoned—a move now forced upon us by both no-go theorems and experiment—the core insight behind Einstein's statistical interpretation can be retained and developed. In particular, although Einstein's *naïve* attempt to complete quantum mechanics locally with non-contextual hidden variables is impossible, this does not undermine the more fundamental claim that the wavefunction alone does not specify the complete physical state.

I argue that, when locality is relinquished, the de Broglie–Bohm pilot-wave theory emerges as the natural successor to Einstein's statistical interpretation. Pilot-wave theory completes quantum mechanics with a spatiotemporal ontology, accommodates contextuality and fundamental nonlocality, avoids macroscopic superpositions via effective collapse, and remains effectively local at the macroscopic scale. In this sense, it realizes—under constraints Einstein himself could not yet have known—the reductionist project that motivated Einstein's critique of quantum mechanics from the start. The remainder of the paper develops and defends this claim.

The paper is organized as follows. In Section 2, I reconstruct Einstein's case for the incompleteness of quantum mechanics by identifying four distinct but related arguments: the non-physical field argument, the ontic probability argument, the nonlocality argument culminating in EPR, and the macroscopic superposition argument associated with the

measurement problem. Section 3 examines the main strategies developed to address these difficulties, with particular attention to approaches that reject or reinterpret collapse, and situates Einstein's statistical interpretation within this broader landscape. In Section 4, I analyze Einstein's statistical interpretation in detail, emphasizing its grounding in classical statistical mechanics and clarifying its disagreement with Born over the status of quantum probability. Section 5 reviews the relevant impossibility theorems—von Neumann's, Bell's, and the Bell–Kochen–Specker theorem—and shows why Einstein's original project of locally completing quantum mechanics with non-contextual hidden variables cannot succeed. Section 6 introduces the de Broglie–Bohm pilot-wave theory and argues that it provides a non-naïve statistical interpretation of the wavefunction by completing quantum mechanics with a spatiotemporal ontology, while accommodating contextuality and fundamental nonlocality. I conclude in Section 7 by arguing that, once locality is abandoned, pilot-wave theory best satisfies Einstein's reductionist aims and offers the most faithful realization of his original explanatory project.

2. Einstein's Arguments for Incompleteness

Bohr held that quantum theory cannot give a complete description of reality because classical concepts fail at the atomic level. For him, this incompleteness is unavoidable and fundamental, motivating an antirealist stance. Einstein agreed that quantum mechanics is incomplete but rejected Bohr's claim that this limitation is fundamental. He understood incompleteness as temporary and remediable, which led him to endorse a statistical interpretation of the wavefunction.

Between 1926 and 1936, Einstein advanced four main arguments for incompleteness: (1) the non-physical field argument, (2) the ontic probability argument, (3) the nonlocality argument culminating in EPR, and (4) the macroscopic superposition argument later known as the measurement problem.

2.1 Failure of Classical Reductionism: The Non-Physical Field Argument

Einstein's first argument rests on classical reductionism, which assumes that matter exists in spacetime and that macroscopic objects are composed of microscopic constituents. This framework enables explanations of macroscopic properties—such as temperature, entropy, and material behavior—in terms of microscopic dynamics, as in statistical mechanics.

Schrödinger's formulation of quantum mechanics represents physical states by a wavefunction evolving via the Schrödinger equation. While this invites a realist interpretation of the wavefunction as a physical field, Schrödinger immediately recognized that for many-particle systems the wavefunction does not reside in spacetime but in a high-dimensional configuration space. Lacking spatiotemporal constituents, macroscopic objects cannot be constructed, and classical reductionism fails. Quantum mechanics therefore appears incomplete unless supplemented with a spatiotemporal ontology.

Realist physicists—including de Broglie, Einstein, Lorentz, and Schrödinger—rejected the many-particle wavefunction as fundamental ontology once it became clear that it could not be interpreted as a physical field in three-dimensional space (de Broglie 1927; Lorentz in Prizbram 1967; Einstein in Howard 1990; Schrödinger in Bacciagaluppi and Valentini 2010). This reaction reflected a shared commitment to constructive explanation, famously articulated in Einstein's distinction between principle and constructive theories, as discussed in section 4 (Einstein 1919).

Because classical reductionism was taken for granted, realists interpreted the non-spatiotemporal character of the wavefunction as evidence that quantum mechanics is incomplete, whereas antirealists saw it as undermining realism itself.

2.2 Failure of Constructive Explanation: The Ontic Probability Argument

A second argument emerges from Einstein's disagreement with Born over the interpretation of quantum probability. Born proposed that the squared modulus of the wavefunction gives objective probabilities for individual measurement outcomes (Born 1926), treating these probabilities as irreducible. Einstein accepted the empirical success of Born's rule but rejected its interpretation.

Einstein's objection can be reconstructed as follows. If quantum mechanics were complete and the wavefunction fully described individual systems, then the probabilities appearing in the Born rule would have to be ontic. Such irreducible probabilities cannot be grounded in a spatiotemporal micro-ontology and therefore fail to provide a constructive explanation of physical phenomena. By contrast, probabilities are unproblematic when they arise epistemically, as in classical statistical mechanics, where they reflect ignorance of underlying deterministic variables.

On Einstein's statistical interpretation, the wavefunction does not describe the real physical state of an individual system but characterizes an ensemble of similarly prepared systems. Quantum probabilities therefore signal incompleteness rather than fundamental indeterminism. As Einstein emphasized in his correspondence with Born, treating probability as a basic element of physical theory amounts to abandoning the goal of explaining phenomena in terms of definite physical states evolving in spacetime (Born 1971).

2.3 Failure of Locality: The EPR Argument

Einstein's most influential argument for incompleteness concerns locality. If quantum mechanics were complete, he argued, it would entail nonlocal interactions. This concern appeared already in Einstein's early discussions of diffraction and was later sharpened through a series of thought experiments, culminating in the Einstein–Podolsky–Rosen (EPR) argument of 1935.

EPR considered two particles in an entangled state such that a measurement performed on one particle allows one to predict with certainty the outcome of a corresponding measurement on

the distant particle. Assuming locality—that measurements cannot instantaneously influence distant systems—EPR concluded that the distant particle must already possess definite properties not specified by the wavefunction. Quantum mechanics is therefore incomplete.

Einstein regarded locality as essential for causal explanation and experimental practice, since without it systems could not be treated as approximately isolated. For this reason, he concluded that any theory predicting nonlocal interactions must be incomplete.

Although these arguments were widely misunderstood—often as challenges to the uncertainty principle (Howard 2004; Norsen 2017; Bricmont 2016)—their core aim was to show that completeness together with locality leads to contradiction.

2.4 Failure of Empirical Adequacy: The Measurement Problem

When the preceding arguments failed to persuade antirealists, realists adopted a different strategy: they argued that quantum mechanics is problematic even by antirealist standards, because it fails to be empirically adequate. This line of reasoning led to the macroscopic superposition argument, later known as the measurement problem.

If a system is completely described by a linearly evolving wavefunction, superpositions propagate from the microscopic to the macroscopic level, leading to unobserved macroscopic superpositions such as Schrödinger's cat (1935a). Since such superpositions are never observed, quantum mechanics appears empirically inadequate unless modified or completed.

Although the measurement problem had a much greater impact on the physics community than Einstein's earlier arguments, it was of limited relevance to his own project. Einstein sought a microscopic explanation of quantum phenomena, whereas many responses to the measurement problem aim merely to suppress macroscopic superpositions without providing a micro-ontology.

Taken together, Einstein's arguments suggest that quantum mechanics lacks a spatiotemporal ontology, fails to provide a constructive explanation of probability, threatens locality, and predicts unobserved macroscopic superpositions. Einstein rejected collapse solutions as nonlocal and regarded both particle and wave ontologies as problematic under the assumption of locality and linear dynamics. Faced with this impasse, he spent the final decades of his life pursuing unified classical field theories, motivated by his commitment to spatiotemporality and locality. Despite their ambition, these efforts ultimately failed (Sauer 2014).

3 Attempts to 'Fix' Quantum Mechanics

Einstein did not know how quantum mechanics should be completed, but he remained convinced that it was incomplete. He nevertheless held a clear view about how to interpret the theory as it stood, centered on his statistical interpretation, discussed in the next section. On this view, the wavefunction does not describe the real physical state of an individual system but

rather an ensemble of similarly prepared systems, closely analogous to classical statistical mechanics. Just as thermodynamics reflects ignorance of underlying molecular states, quantum probabilities express ignorance of deeper variables that fully determine physical reality. Einstein thus accepted the empirical success of quantum mechanics, including Born's rule, while denying its completeness and insisting on realism, locality, and separability.

Einstein was largely alone in holding this position. Of the problems discussed in Section 2, the measurement problem attracted by far the most attention: the non-physical field argument was never explicitly articulated, and the EPR argument was widely misunderstood. By contrast, the measurement problem was too striking to ignore and prompted a range of responses. In what follows, I briefly survey the most influential of these to highlight their contrast with Einstein's approach.

3.1 'Physical' Collapse

Traditionally, the measurement problem is solved by introducing von Neumann's collapse postulate (1932): when a measurement is performed, the wavefunction randomly and instantaneously collapses to one of the terms of the superposition. Quite apart from the fact it produces a nonlocal action, it was never a satisfactory option for realists, since it leaves unexplained what physically happens during observation and why measurement should differ so radically from other physical interactions as to induce a sudden collapse of the wavefunction.

A more realist-friendly proposal was advanced by Wigner (1961), who suggested that observation is a special process involving not only interactions between material systems but also the interaction between matter and consciousness. On this view, consciousness itself brings about collapse. However, this approach implies that the physical world is not causally closed, requires a theory of consciousness to do physics, and entails that in the absence of conscious observers the world would remain in a superposition of macroscopic states. Given these consequences, it is natural to explore less radical alternatives.

One such alternative treats collapse as embedded in the dynamics itself. On this approach, there is no separate collapse postulate; instead, the Schrödinger equation is taken to break down spontaneously and randomly. The fundamental dynamics is no longer linear and deterministic but includes rapid spontaneous localization of macroscopic systems, ensuring that macroscopic superpositions are never observed. This is the strategy adopted by spontaneous localization theories, first proposed by Ghirardi, Rimini, and Weber (1986). Although less radical than invoking consciousness, these theories arguably remain somewhat phenomenological, providing a precise recipe for matching experimental data while leaving open the deeper question of what physically triggers collapse.

3.2 Collapse and Incompleteness

A very different response to the measurement problem is to deny that wavefunction collapse is a physical process at all. On this view, the wavefunction encodes incomplete information about

a system, and collapse is merely an update of information rather than a physical change. In the Schrödinger cat scenario, the superposed wavefunction reflects ignorance of whether the cat is alive or dead; opening the box simply reveals the cat's pre-existing state, prompting an update of the wavefunction without altering the cat itself. The wavefunction is thus epistemic rather than ontic.

Interpreting collapse as an information update, however, amounts to conceding that quantum theory is incomplete. If measurement reveals information not contained in the pre-measurement wavefunction, then a Schrödinger-evolving wavefunction does not fully describe physical reality. In this sense, epistemic accounts of collapse implicitly accept incompleteness.

Bohr's position can be understood along these lines. He argued that we describe quantum systems as we do because our classical concepts are inadequate (Bohr 1928), but he further maintained that this limitation is fundamental rather than temporary. By contrast, one could adopt an epistemic view of collapse while still hoping for a future completion of quantum mechanics. Unlike Einstein, however, the Copenhagen interpretation abandons realism about microscopic properties and treats indeterminacy as intrinsic.

Contemporary approaches such as QBism or Quantum Bayesianism (Fuchs 2011a,b; Fuchs, Mermin, and Schack 2014) and Healey's pragmatist quantum realism (Healey 2012), often grouped under the label "neo-Copenhagen," also adopt an epistemic view of the wavefunction. In these accounts, the wavefunction represents an agent's information, and collapse reflects the acquisition of new information through measurement rather than a physical process. While some of these approaches allow for a future completion of quantum mechanics, others treat incompleteness as intrinsic and unavoidable.

4. The Statistical Interpretation

Einstein's position was closer to Bohr's than to Wigner's or to spontaneous-collapse approaches. Like Bohr, he held that quantum mechanics is incomplete, but unlike Bohr he rejected the idea that this incompleteness is fundamental or unavoidable. Even though he did not know how to complete the theory, Einstein was convinced that quantum mechanics is provisional and would eventually emerge from a deeper framework. For him, incompleteness meant the absence of a microscopic description—precisely the concern expressed by the non-physical field argument.

Bohr, by contrast, came to believe that a microscopic description was impossible, a view that led him toward an antirealist position. Once one abandons realism about microscopic structure, concerns about the non-physical field and locality lose their force, and the EPR argument ceases to be troubling. What remains is the measurement problem, which can then be addressed at the macroscopic level without supplying a microscopic account.

Einstein, however, insisted on a microscopic story, and quantum theory provided none. This demand was not radical but deeply rooted in the realist tradition, where realism and

microscopic reductionism go hand in hand. Before quantum theory, fundamental physics—classical mechanics—successfully reduced higher-level theories such as rigid body mechanics, hydrodynamics, and thermodynamics to microscopic laws. The most difficult case was thermodynamics, which Boltzmann ultimately grounded in statistical mechanics.

This outlook fits naturally with Einstein's distinction between principle and constructive theories. Principle theories impose general constraints derived from empirical regularities, whereas constructive theories explain those regularities in terms of underlying microscopic structures and dynamics. Although principle theories are useful, Einstein regarded them as incomplete without a constructive foundation. On this view, quantum mechanics at best resembles thermodynamics before the advent of statistical mechanics: a powerful principle theory still awaiting a microscopic foundation. (The qualification "at best" reflects the fact that quantum mechanics does not even reach that level, since it deals only with laboratory measurements.)

4.1 The Analogy with Classical Statistical Mechanics

Einstein's statistical interpretation of the wavefunction is best understood by analogy with classical statistical mechanics. Convinced that quantum theory is incomplete but completable, Einstein held that the wavefunction does not describe the physical state of an individual system but characterizes an ensemble of identically prepared systems. This is the core of his statistical interpretation. A complete theory, by contrast, would provide a microscopic description of individual systems; precisely because quantum mechanics lacks such a description, its formalism applies only at the ensemble level.

For Einstein, this incompleteness lay behind several central difficulties of quantum theory, including the pervasive role of probabilities, apparent nonlocality, and macroscopic superpositions. Probability itself was not problematic, provided it was not fundamental. In classical statistical mechanics, probabilities arise because systems are too complex for their microscopic states to be fully known. A gas consists of countless molecules with definite positions and momenta, whose collective behavior is described statistically only because of practical ignorance; the underlying dynamics remain deterministic.

Einstein argued that quantum probabilities should be understood in the same way. While accepting the mathematical validity of Schrödinger's equation, he denied that the wavefunction provides a complete description of an individual system. Instead, it summarizes incomplete information about deeper variables that fully determine physical reality. Just as macroscopic quantities such as temperature and pressure do not describe individual molecules, the wavefunction does not represent the real physical state of a single particle.

This ensemble interpretation reflects Einstein's broader commitments to realism and locality. Physical systems, he insisted, must possess definite properties independently of observation, and probabilities signal the omission of relevant variables—hidden from the current

formalism—that determine individual outcomes. The wavefunction thus plays a role analogous to a probability distribution in statistical mechanics.

Einstein emphasized that such a situation would not be unprecedented. Thermodynamics was once regarded as complete but later understood as a statistical approximation to underlying molecular dynamics. In the same spirit, he viewed quantum mechanics as a provisional and effective theory, awaiting a deeper foundation.

4.2 The Born Rule

Einstein's statistical interpretation took shape through his sustained dialogue—and eventual disagreement—with Born. In 1926, Born proposed that the wavefunction should be understood as a probability amplitude, with $|\psi(x)|^2$ giving the probability density for finding a particle at position x (Born 1926). Born held that this probabilistic character is irreducible: the wavefunction provides the most complete possible description of an individual system.

Einstein immediately acknowledged the empirical success of Born's rule and accepted it as mathematically correct. However, he rejected Born's interpretation of what the probabilities signify. In a 1926 letter to Born, Einstein insisted that "the ψ-function does not describe a state of one system but rather a statistical ensemble of many systems."

The disagreement was therefore not about predictive success, but about completeness. Born regarded quantum probability as fundamentally different from classical statistical probability, denying the existence of any deeper microphysical description underlying the wavefunction. Einstein, by contrast, insisted that quantum probabilities should be understood in strict analogy with classical statistical mechanics: just as thermodynamic probabilities arise from averaging over unknown molecular states, quantum probabilities arise from averaging over variables not represented in the theory.

Einstein's well-known remark—"God does not play dice" (Born 1971)—should be read in this light. It expresses not a rejection of probability as such, but a rejection of Born's claim that probability is ultimate. Einstein accepted Born's statistical rule while denying that it applies to individual systems.

4.3 Hidden Variables

Einstein's statistical interpretation also motivated the introduction of hidden variables. What is missing from quantum mechanics, on this view, is a spatiotemporal element that restores locality and removes macroscopic superpositions as a consequence rather than as a primary aim. The complete microscopic state would therefore include additional variables—denoted by λ—alongside the wavefunction. Einstein did not specify the nature of these variables, partly because he recognized how difficult completing quantum theory would be.

The original EPR argument was formulated in terms of position and momentum. EPR considered a two-particle state with perfect correlations: if particle A has position x, particle B

has position $x + c$; if A has momentum p, B has momentum $-p$. Assuming locality, if measuring A allows one to predict with certainty the outcome of a measurement on B, then B must already possess that property. These pre-existing values are the hidden variables required to explain the correlations without invoking nonlocality.

The use of both position and momentum led to misunderstandings, particularly the mistaken idea that the argument conflicts with the uncertainty principle. As Einstein emphasized, the uncertainty principle concerns measurement limitations, not the existence of properties (Fine 1986).

A clearer formulation was later provided by Bohm (1951) using a spin-singlet state. Explaining the perfect anticorrelations locally requires assuming that the spin values were already determined and merely revealed by measurement, reinforcing the idea that the entangled wavefunction is incomplete (Norsen 2017).

4.4 Comparison with Other Incompleteness Approaches

Einstein's statistical interpretation contrasts sharply with other approaches that accept quantum incompleteness. Bohr rejected the reductionist ideal altogether, treating classical and quantum descriptions as mutually complementary and irreducible. Pragmatist and neo-Copenhagen approaches similarly deny that quantum theory aims to represent microscopic reality, instead treating the formalism as a tool for organizing expectations.

QBism radicalizes this stance by locating quantum probabilities in the beliefs of individual agents. Measurement outcomes do not reveal pre-existing properties but arise through agent–world interactions. Quantum theory thus provides rules for expectation management rather than a description of physical reality.

Although these approaches are often labeled epistemic (Maudlin 2019), the statistical interpretation differs importantly from them. On Einstein's view, the wavefunction does not represent the material ontology of a system; rather, the complete state is given by the wavefunction together with additional variables specifying the system's physical properties. The wavefunction alone characterizes an ensemble, not an individual system. Our ignorance arises not because the wavefunction represents subjective knowledge, but because it omits part of the objective state.

For this reason, the wavefunction in the statistical interpretation is not epistemic in the neo-Copenhagen sense. Although incomplete, it represents an objective feature of the system (see also Hubert 2023). This distinction also explains why the statistical interpretation does not face the standard "problem of interference" often raised against epistemic views. Interference can still be explained by the dynamics of the underlying ontology guided by the wavefunction—a point developed further in connection with pilot-wave theory.

Finally, as we will see in the next section, although impossibility theorems show that measurement outcomes cannot be understood as simply revealing pre-existing values, this does not rule out a microscopic description altogether. In section 6, I argue that the de Broglie–Bohm pilot-wave theory provides a non-naïve version of the statistical interpretation compatible with these results.

5. Impossibility Theorems

Unfortunately, this optimistic picture is misleading: Einstein's statistical interpretation does not work. Von Neumann (1932) proved a theorem long taken to show that quantum mechanics cannot be completed by a precise microscopic description of the kind Einstein envisioned, since any such completion would fail to reproduce the theory's empirical predictions. Although von Neumann's theorem was later shown to rely on overly restrictive assumptions (see Hermann 1935 and Bell 1966), other no-go results were subsequently established that avoid these shortcomings, including the Kochen–Specker theorem and Bell's own theorem.

In this section, I argue that these results—together with a largely overlooked remark by Schrödinger—undermine Einstein's statistical interpretation. Schrödinger's observation shows that the hidden variables required by Einstein's approach, if they are to respect locality, must satisfy specific mathematical constraints. The later no-go theorems then show that no set of hidden variables satisfying these constraints can reproduce the quantum predictions.

Because of von Neumann's authority, along with historical and sociological factors, many physicists abandoned the search for such completions. Nevertheless, as argued in the final part of this paper, pilot-wave theory remains the only known completion of quantum mechanics that fully reproduces its empirical predictions while satisfying all of Einstein's demands except locality. Moreover, the nonlocality involved is of a special kind that does not manifest at the macroscopic level and is therefore far less problematic than Einstein originally feared.

5.1 Schrödinger's Non-contextual Value Maps

As discussed in connection with EPR, Einstein, Podolsky, and Rosen held that—assuming locality—the only way to explain perfect anticorrelations is to treat measurement outcomes as revealing pre-existing properties of the system. This requires supplementing the wavefunction with additional variables, the so-called hidden variables, which complete the description provided by quantum mechanics. The resulting theory must be local (since Einstein rejected nonlocal influences) and "hidden" because quantum mechanics specifies only the wavefunction and remains silent about the additional properties.

In quantum theory, observables are represented by self-adjoint operators, and possible outcomes correspond to eigenvalues. Explaining perfect (anti)correlations for an observable Atherefore requires that each particle possesses a corresponding property with values $H_A =$

$v(A)$, where $v(A)$is an eigenvalue of A. A function $v: A \mapsto v(A)$assigning values to observables is known as a value map (Bricmont et al. 2020a).

Schrödinger (1935b,c; 1936; see also Bricmont et al. 2020a,b, 2022) observed that not just any hidden variables can explain the EPR correlations locally. He showed that such variables must satisfy the same functional relations as the (mutually commuting) operators representing the corresponding observables. In particular, if observables A, B, and Ccommute and satisfy $C = f(A, B, \dots)$, then the assigned values must satisfy $v(C) = f(v(A), v(B), \dots)$. For example, if $C = A + B$, then $H_C = H_A + H_B$.

This requirement later became known as the non-contextuality condition. It expresses the idea that outcomes reveal properties whose values are independent of the measurement context—namely, independent of which other commuting observables are measured alongside them or in what order. Schrödinger's crucial point, often overlooked, is that this condition is not an optional assumption: it is forced upon any *local* hidden variable theory that aims to explain the EPR correlations. If the condition failed, there would be no well-defined properties capable of accounting for the correlations without invoking nonlocal influences.

Although the formalism may appear abstract, the underlying idea is simple: to explain correlations locally, outcomes must correspond to pre-existing properties; and for such properties to be well-defined, their values must be independent of measurement context. In this sense, the statistical interpretation amounts to understanding the wavefunction as part of a local, non-contextual hidden variable theory—one in which the wavefunction is incomplete and must be supplemented by additional variables whose values obey the same relations as the corresponding commuting observables.

5.2 Bell and Kochen–Specker's Theorems

We can now move to the second step: the kind of hidden variables required—namely, local and non-contextual—cannot reproduce the quantum predictions. As anticipated, the first result along these lines was due to von Neumann, who argued that if $C = A + B$, then it is impossible for the corresponding values to satisfy $v(C) = v(A) + v(B)$. However, von Neumann's proof relied on relations between non-commuting operators, which have no joint eigenvalues and cannot be measured simultaneously; the assumed relations between their values therefore lack physical meaning.

Subsequent no-go theorems avoided this flaw by restricting attention to mutually commuting operators. In particular, Bell (1966), and independently Kochen and Specker (1967), proved what is now often called the Bell–Kochen–Specker (BKS) theorem. These results show that non-contextual hidden variable theories—whether local or not—are impossible. They do so by exhibiting sets of commuting observables for which the functional relations required by non-contextuality cannot be satisfied by any assignment of values. That is, there are cases in which,

even though quantum mechanics implies $C = f(A, B, \dots)$, it is impossible for the corresponding values to satisfy $v(C) = f(v(A), v(B), \dots)$.

This conclusion is directly relevant to Einstein's program. As Schrödinger had already shown, non-contextual hidden variables are precisely what is required to explain the EPR correlations while preserving locality. The no-go theorems therefore imply that no such completion of quantum mechanics can reproduce the quantum predictions. Since those predictions are empirically confirmed, local non-contextual hidden variable theories—and thus the statistical interpretation in its naïve form—are empirically inadequate.

5.3 Nonlocality

Taken together, the BKS result and the EPR conclusion yield a proof of nonlocality that does not rely on Bell inequalities (Bricmont et al. 2020b). In Schrödinger's generalized version of EPR, locality implies the existence of non-contextual hidden variables; the BKS theorem then shows that non-contextual hidden variables are incompatible with the predictions of quantum mechanics. The only remaining conclusion is that the locality assumption must be false.

Traditionally, nonlocality is derived from the violation of Bell inequalities. Bell's reasoning begins from the EPR premise: if quantum mechanics is to be locally completed, it must involve hidden variables, denoted generically by λ. Bell showed that any such local completion satisfies a specific inequality, whereas quantum mechanics predicts violations of it. Since experiments—beginning with those of Aspect et al. (1981)—confirm the quantum predictions, the local hidden-variable completion is empirically ruled out: locality implies the Bell inequality; experiments violate the inequality; therefore locality is false.

Historically, Bell's work was motivated by Bohm's 1952 theory, discussed in the next section, which, by supplementing the wavefunction with particles positions, constituted a counterexample to von Neumann's theorem. Bell famously remarked that in Bohm's theory he had seen "the impossible done." Bohm's theory is explicitly nonlocal, but shows that it is possible to complete quantum theory at least in principle. Bell then asked himself whether a local completion was possible and returned to EPR to investigate this question. His inequality revealed that local completions are empirically distinguishable from quantum mechanics—and experimentally falsified. The conclusion is that any viable completion of quantum theory must be nonlocal.

5.4 Contextuality

Once locality is dropped—as it must be—the mapping between observables and values must be contextual. It cannot be non-contextual, since the impossibility of non-contextual hidden variable theories proven by Bell and by Kochen and Specker holds independently of locality. If non-contextual hidden variables are ruled out in general, any viable completion of quantum theory must involve contextuality.

Contextuality means that the value associated with an observable depends on the measurement context: the specific experimental procedure implementing the measurement, or which other compatible observables are measured alongside it. The same operator can yield different outcomes depending on the arrangement that realizes its measurement.

There are two ways to understand this dependence. One is to insist that outcomes reveal genuine properties of the system, but to accept that these properties are unusual in that their values vary with context. The other is to deny that outcomes represent intrinsic properties at all, interpreting them instead as features of the interaction between system and apparatus. On this view, outcomes depend on context because they depend on the physical coupling involved in the measurement.

The second option is more compelling. If outcomes vary with context, this suggests that measuring an observable is not, in general, revealing a pre-existing property. Rather, measurement is a physical process that can substantially disturb the system, so the observed value $v(A)$cannot be identified with a value the system possessed prior to the interaction. This way of thinking aligns with Heisenberg's insight that measurement necessarily involves disturbance: for example, determining the position of an electron requires illuminating it with light, but the collision between photon and electron inevitably changes the electron's position (Daumer et al. 1996). In this light, many quantities represented by operators do not correspond to properties of the system in the classical sense.

This conclusion supports the QBists' claim that in quantum experiments we actively participate in reality, in the sense that our interventions unavoidably change the systems we study (see the 2016 Fuchs' participatory realism paper). However, acknowledging this does not require accepting the stronger QBist conclusion that a deeper microscopic understanding of quantum phenomena is impossible. As argued in the next section, the fact that measurements do not reveal pre-existing properties does not by itself rule out an underlying account of quantum processes.

6. The Pilot-wave Theory

The pilot-wave approach to quantum theory has its roots in Einstein's early attempts to understand wave–particle duality, even before the formal development of quantum mechanics. Einstein hoped that interference phenomena involving particle-like photons could be explained if their motion were guided by the electromagnetic field, conceived as a guiding field (Bacciagaluppi and Valentini 2010). Although this idea proved problematic, it introduced a lasting intuition: particles might follow definite trajectories while being guided by a wave-like structure.

Soon after, Born explored the possibility that the wavefunction itself could play the role of a guiding field or "pilot wave" (Born 1926), though this suggestion met resistance from Heisenberg. The decisive step was taken by Louis de Broglie, whose 1924 doctoral thesis

proposed a dual ontology of particles and waves. In 1927, after Schrödinger's discovery of wave mechanics, de Broglie derived an explicit equation of motion for particles guided by a scalar wavefunction and showed how this dynamics could account for quantum interference (de Broglie 1927). He presented this theory at the 1927 Solvay Conference.

Despite its promise, de Broglie's proposal was quickly abandoned, in part due to Pauli's objection concerning inelastic scattering (Pauli 1928), which de Broglie did not successfully answer (Bacciagaluppi and Valentini 2010). Born and de Broglie soon aligned themselves with the Copenhagen consensus, while Schrödinger pursued—and ultimately abandoned—a pure wave ontology, which leads to macroscopic superpositions even when projected into three-dimensional space.

Pilot-wave theory was independently rediscovered by Bohm in 1952, who developed it into a coherent alternative to orthodox quantum mechanics. Einstein acknowledged that Bohm had shown such a completion to be logically possible, but famously dismissed the theory as "too cheap," objecting to its explicit nonlocality and lack of a unified field-theoretic basis (Born 1971). John Bell later became its most influential advocate, emphasizing its significance as a counterexample to claims—stemming from von Neumann—that hidden-variable completions were impossible.

In its modern form—often called de Broglie–Bohm, pilot-wave theory or Bohmian mechanics—the theory posits particles with definite trajectories guided deterministically by the wavefunction. Writing $\psi = Re^{iS}$, particle velocities are given by the guiding equation $v = \nabla S/m$. If particle configurations are initially distributed according to the quantum-equilibrium distribution $|\psi|^2$, the theory reproduces the empirical predictions of standard quantum mechanics. Bohm's alternative second-order formulation using a "quantum potential" is generally regarded as an artificial reformulation of this fundamentally first-order dynamics (Dürr, Goldstein, and Zanghì 1992).

Because pilot-wave theory is formulated for the universe as a whole, it employs conditional and effective wavefunctions to describe subsystems. Under typical conditions—especially measurement interactions combined with decoherence—the effective wavefunction evolves autonomously and mimics collapse: although the universal wavefunction never literally collapses, only one effective branch guides the actual configuration, while the others become dynamically irrelevant.

Pilot-wave theory is explicitly nonlocal: when the wavefunction is entangled, particle velocities depend instantaneously on distant configurations. This nonlocality is not an optional add-on but is what allows the theory to reproduce violations of Bell inequalities while remaining deterministic. At the same time, the theory entails a fundamental epistemic limitation. Measurement interactions disturb the guiding wavefunction in ways that prevent access to the exact particle configuration, and equivariance ensures that configurations distributed according

to $|\Psi|^2$ remain so. Observable predictions are therefore probabilistic, reflecting ignorance of the actual configuration rather than indeterminism in the laws.

It has nevertheless been argued that pilot-wave theory may yield empirically distinguishable predictions in specific contexts—most notably in time-of-arrival experiments (Das and Dürr 2019; for criticism see Goldstein et al. 2024a,b). Quantum equilibrium also screens nonlocality at the operational level, preventing superluminal signaling (Eberhard 1978). Valentini (1991) has emphasized this role and later suggested the possibility of quantum non-equilibrium (Valentini 2010), though at present we lack a clear account of how such deviations could be detected.

6.1 Back to Einstein

Returning to Einstein, his overarching aim was to provide a spatiotemporal ontology of matter, account for probability, preserve local interaction, and ensure empirical adequacy. Ultimately, he sought a microscopic, reductive explanation of quantum phenomena. He hoped to achieve this by supplementing quantum mechanics with spatiotemporal, non-contextual hidden variables, rendering probabilities epistemic and preserving locality. How macroscopic superpositions were to be suppressed remained unclear, though Einstein hoped a future—presumably nonlinear—theory would resolve this difficulty.

As anticipated, Einstein rejected pilot-wave theory primarily because it is explicitly nonlocal. From his standpoint, this was decisive. Moreover, pilot-wave theory is contextual—something that disqualified it as a *local* completion of the EPR kind. From our current perspective, however, this assessment changes. We now know that locality is false and that non-contextuality is untenable: any empirically adequate theory must be contextual.

Pilot-wave theory supplements the wavefunction with a spatiotemporal ontology given by the actual particle configuration Q, so that the complete state is $S = (Q, \psi)$. The wavefunction is not epistemic in the neo-Copenhagen sense, but an objective component of the state governing motion. Measurement outcomes are contextual, but this is unavoidable: non-contextual value maps are ruled out by the Bell–Kochen–Specker theorem independently of locality.

Accordingly, pilot-wave theory does not treat measurement outcomes as straightforward revelations of pre-existing values for arbitrary observables. Outcomes depend on the concrete interaction between system and apparatus; a "measurement of momentum," for example, need not reveal a prior momentum value but yields an outcome fixed by the interaction dynamics (see Bricmont 2016). In this sense, value assignments do not represent intrinsic properties of the system prior to measurement.

Somewhat ironically, pilot-wave theory is the only hidden-variable theory in which hidden variables are *not* introduced to explain perfect (anti)correlations locally. Those correlations are explained by nonlocal dynamics. This is precisely why the theory succeeds, whereas all local hidden-variable strategies fail.

6.2 Quantum Classical Reductionism

Even though pilot-wave theory is nonlocal and contextual, it remains the most promising option for a reductionist in Einstein's sense. It provides a three-dimensional ontology of matter—particles with definite positions—and supports a compositional, statistical-mechanical style of reduction, in which macroscopic objects are built from microscopic constituents whose dynamics explain higher-level regularities (see Allori forthcoming).

Einstein's methodological worry about nonlocality concerned isolation: if distant influences matter, how can systems be treated as approximately independent? Pilot-wave nonlocality, however, is effectively suppressed at the macroscopic scale. Decoherence, namely the interaction of the system with its environment, rapidly stabilizes autonomous dynamics for macroscopic subsystems, rendering nonlocal dependencies operationally irrelevant in ordinary contexts. This is sufficient for reductionist explanation.

Pilot-wave theory also avoids macroscopic superpositions. Because matter is represented by particle configurations, macroscopic objects are never in superpositions of distinct configurations. Decoherence yields an effective collapse: although the universal wavefunction evolves unitarily, only one effective wavefunction guides the actual configuration, producing definite outcomes without a fundamental collapse postulate.

Contextuality does not undermine reductionism. The Bell–Kochen–Specker theorem shows that contextuality is unavoidable, and many quantities we treat as properties—such as energy or momentum—can be understood as emergent or interaction-dependent without threatening reductionist explanation, much as thermodynamic variables are emergent despite being explainable from microdynamics (see Allori forthcoming).

The remaining tension with relativity reflects not a failure of pilot-wave theory, but the difficulty of reconciling quantum nonlocality with relativistic structure. One may argue that this suggests the need to supplement relativity itself with additional structure, such as a preferred foliation, as in pilot-wave theory. Quantum mechanics is incomplete because it cannot accommodate spacetime; relativity is incomplete because it cannot account for nonlocality. Adding particles to the former and a preferred foliation to the latter, which is what the pilot-wave theory does, seems to be the best way to combine them (Allori 2024).

Finally, the analogy with classical statistical mechanics reinforces this picture. In both cases deterministic microdynamics coexist with probabilistic macroscopic behavior due to typicality and epistemic limitations (Dürr et al. 1992). Just as thermodynamics emerges from particle mechanics, quantum statistics emerge in pilot-wave theory. This parallel was central to Einstein's thinking and explains why pilot-wave theory—despite nonlocality and contextuality—best realizes his reductionist ambitions.

7. Summary and Conclusion

Einstein sought to complete quantum mechanics in a local way and to ground it in spacetime, since only Einstein sought to complete quantum mechanics by grounding it in a spatiotemporal ontology capable of sustaining the kind of reductive explanation he valued. Within this framework, the wavefunction is naturally interpreted statistically: it characterizes ensembles of similarly prepared systems rather than the physical state of an individual system.

Einstein hoped to achieve such a completion by introducing non-contextual hidden variables. However, the BKS theorem shows that this strategy cannot succeed: any completion of quantum mechanics must be contextual. Moreover, as Schrödinger had already emphasized, non-contextual hidden variables are required only if one insists on explaining the EPR correlations locally. Once locality is abandoned, that requirement disappears.

And locality must indeed be abandoned. The EPR–Bell–Aspect results, together with the EPR–Schrödinger–BKS line of reasoning, show that nature is fundamentally nonlocal. It is therefore pointless to pursue a local completion of quantum mechanics. What remains essential, however, is to complete the theory with a spatiotemporal ontology—a requirement Einstein likely regarded as so obvious that it did not need explicit defense.

Pilot-wave theory accomplishes precisely this by introducing particles with definite positions, thereby turning quantum mechanics into a theory of matter in spacetime. It is necessarily contextual, as required by the BKS theorem: observables do not correspond to intrinsic properties of systems, but to the outcomes of concrete physical interactions between the system and the measuring apparatus. In this framework, the only quantity that is genuinely measured is particle position—unsurprising, given that measuring devices are themselves composed of the same particles they probe.

Pilot-wave theory is also nonlocal, a feature Einstein strongly resisted. Yet nonlocality is unavoidable, given the available no-go theorems. Crucially, however, this nonlocality is effectively suppressed at the macroscopic scale, so that it does not undermine the autonomy of macroscopic systems or the possibility of successful reductionist explanation.

In light of these considerations, pilot-wave theory satisfies all of Einstein's substantive demands except locality—and locality, we now know, cannot be satisfied. Had Einstein been aware of the BKS theorem and of the later experimental confirmations of nonlocality (Bell, Aspect), he might well have regarded pilot-wave theory not as "too cheap," but as the only viable completion of quantum mechanics consistent with his deepest methodological commitments.

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